



1	Application of particle swarm optimization for gravity inversion of 2.5-D
2	sedimentary basins using variable density contrast
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6	Abstract
7	Particle swarm optimization (PSO) is a global optimization technique that works similarly to
8	swarms of birds searching for food. A Matlab code in PSO algorithm is developed to estimate
9	the depth to the bottom of a 2.5-D sedimentary basin and coefficients of regional background
10	from observed gravity anomalies. The density contrast within the source is assumed to be
11	varying parabolically with depth. Initially, the PSO algorithm is applied on synthetic data
12	with and without some Gaussian noise and its validity is tested by calculating the depth of the
13	Gediz Graben, Western Anatolia and Godavari sub-basin, India. The Gediz Graben consists
14	of Neogen sediments and the metamorphic complex forms the basement of the Graben. A
15	thick uninterrupted sequence of Permian-Triassic and partly Jurassic and Cretaceous
16	sediments forms the Godavari sub-basin. The PSO results are better than the results obtained
17	by Marquardt method and the results are well correlated with borehole information.
18	Keywords: Particle swarm optimization, Sedimentary basin, Gravity anomaly, Inversion,
19	Gaussian noise.
20	
21	Introduction
22	Gravity method is a natural source method which helps in locating masses of greater or lesser

- 23 density than the surrounding formations. It is used as a reconnaissance survey in hydrocarbon
- exploration. Sedimentary basins, which are characterized by negative gravity anomalies, are 24





25 the location of almost all of the world's hydrocarbon reserves. Interpretation of gravity data is 26 a mathematical process of trying to optimize the parameters of the initial model in order to 27 get a good match to the observed data. Interpretation of gravity data is always associated with 28 the ambiguity problem. Ambiguity in gravity anomalies can be overcome by assigning a 29 mathematical geometry to the anomaly causing body with a known density contrast (Rama 30 Rao and Murthy, 1978). Bott (1960) used stacked prism model to describe the cross-section 31 of a sedimentary basin, whereas Talwani (1959) used the polygonal model to describe source 32 geometry. The parabolic density function is used to remove the complications associated with 33 exponential (Cordell, 1973), cubic (Garcia-Abdeslem, 2005) and quadratic (Gallardo-Delgado et al., 2003) density functions. The Marquardt inversion through residual gravity 34 35 anomalies delineates the structure of a sedimentary basin by estimating regional background 36 (Chakravarthi and Sundararajan, 2007). Several authors have developed 2D/2.5D local 37 optimization techniques over the 2D/2.5D sedimentary basin (Annecchione et al., 2001; Barbosa et. al, 1999; Bhattacharya and Navolio, 1975; Litinsky, 1989; Morgan and Grant, 38 39 1963; Murthy et al., 1988; Murthy, and Rao, 1989; Won and Bavis, 1987) to interpret gravity 40 anomalies with constant density function. In many publications over 3D gravity field computation with an approximation of geological bodies by 3D polygonal horizontal prism 41 42 was applied (Eppelbaum and Khesin, 2004; Khesin et al. 1996). Rao (1990) used a quadratic 43 density function, which is comparatively reliable to analyze gravity anomalies over basins 44 having a limited thickness, whereas Chakravarthi and Rao (1993) have done in modeling and 45 inversion of gravity anomalies with quadratic density function.

46 Particle swarm optimization (PSO) is a robust stochastic optimization technique based 47 on the movement and intelligence of swarms, which was developed by James Kennedy and 48 Russell Eberhart (1995). PSO applies the concept of social optimization in problem solving in 49 various fields. In this paper, a Matlab code based on PSO is developed to interpret the gravity





- 50 anomalies of 2.5-D sedimentary basins, where the density varies parabolically with depth.
- 51 PSO analyzed results are consistent and more accurate with other techniques and also well
- 52 agreement significantly with borehole information.

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54 Theory

- 55 In the Bott's approach, sedimentary basin is approximated by a series of vertical prisms. The
- 56 gravity anomaly g_b at any point on the profile AA' as shown in Figure 1.

57
$$g_{b} = \sum_{j=2}^{N-1} g_{j}(x_{l}, 0) + Cx + D$$
(1)

58 The gravity effect of *l*th prism is given by Chakravarthi and Sundararajan (2006) as

59
$$g(x_{l},0) = \int_{z=z_{l}}^{z_{2}} \int_{y=-a}^{a} \int_{x=-c}^{c} \frac{G\Delta\rho(z)zdxdydz}{\left[\left(x-x_{l}\right)^{2}+y^{2}+z^{2}\right]^{3/2}}$$
(2)

60 The parabolic density function at any depth w is given by

61
$$\Delta \rho(w) = \frac{\Delta \rho_0^3}{\left(\Delta \rho_0 - \alpha w\right)^2}$$
(3)

62 Finally, after integration, Equation (2) becomes

$$63 \qquad g(x_{1},0) = -2G\Delta\rho_{0}^{3} \left\{ \begin{pmatrix} \frac{\alpha x_{1}L}{p_{4}} \left(\frac{1}{p_{2}} + \frac{1}{p_{3}}\right) \ln \frac{p_{5}}{p_{6}} + \frac{L}{2P_{2}} \ln \frac{(K+x_{1})}{(K-x_{1})} + \frac{x_{1}}{2p_{3}} \ln \frac{(K+L)}{(K-L)} + \begin{pmatrix} x_{1}+L \\ \frac{\Delta\rho_{0}}{\alpha} \left[\frac{1}{p_{2}} \tan^{-1} \left(\frac{KL}{zx_{1}}\right) + \frac{1}{p_{3}} \tan^{-1} \left(\frac{x_{1}K}{zL}\right)\right] - \frac{1}{\alpha p_{5}} \tan^{-1} \left(\frac{Lx_{1}}{zL}\right) + \begin{pmatrix} x_{1}+L \\ x_{1}+L \\$$

64 Where,

65
$$K = \left(x_1^2 + L^2 + z^2\right), p_1 = x_1^2 + L^2, p_2 = L^2 \alpha^2 + \Delta \rho_0^2,$$

66
$$p_3 = x_1^2 + \Delta \rho_0^2, p_4 = \sqrt{\left(p_1 \alpha^2 + \Delta \rho_0^2\right)}, p_5 = \Delta \rho_0^2 - \alpha z \text{ and}$$

67
$$p_6 = -2(\alpha K p_4 + p_1 \alpha^2 + \Delta \rho_0 \alpha z).$$





Here, *N* is the number of observations, *G* is the universal gravitational constant, *C* and *D* are coefficients of regional background, *c* is half width of the prism, z_1 and z_2 are depths to the top and bottom of the basin, 2*L* is strike length of the prism, *a* is the offset of profile from the center of the prism and $\Delta \rho_0$ and α are constants of the parabolic density function at depth *z*. Since profile AA' does not pass through the centers of each prism, Equation 4 has to be calculated twice by putting *L-a* and *L+a* for *L* and taking the average. The initial depths of the basin calculated using observed anomaly g_0 , is given by as

$$d_{i} = \frac{g_{0}(x_{i})\Delta\rho_{0}}{41.89\Delta\rho_{0}^{2} + \alpha g_{0}(x_{i})}, i = 2, 3, \dots, N-1$$
(5)

Profile AA' entirely covers the lateral dimensions of the sedimentary basin, therefore the depth of the basin on either side of the profile become zero. So, $d_1 = 0 = d_N$

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79 Particle Swarm Optimization

80 PSO uses a number of particles (solutions) that constitute a swarm, moving around in the 81 search space looking for the best solution. Each particle adjusts its "flying" according to its own flying experience as well as the flying experience of other particles. Each particle keeps 82 83 track of its coordinates in the solution space which is associated with the best solution 84 (fitness) that has achieved so far by that particle. This value is called personal best, Pbest. 85 Another best value that is tracked by the PSO is the best value obtained so far by any particle 86 in the neighborhood of that particle. This value is called global best, Gbest. The basic idea of 87 PSO lies in accelerating each particle towards its Pbest and the Gbest locations with a random 88 weighted acceleration at each time step (Mohapatra and Das, 2013).

89
$$V_t^k = w.V_t^{k-1} + c_1.rand_1.(Pbest_t - X_t^k) + c_2.rand_2.(Gbest - X_t^k)$$
(6)

90
$$X_t^k = X_t^{k-1} + V_t^k$$
 (7)





where k is the number of iterations; t is the particle number; V_t^k is the velocity of the th 91 particle at k iterations; X_t^k is the position of tth particle at k iterations; *Pbest*, is the best 92 position of individual tth particle (Local best position); Gbest is the best position of all 93 94 particles (Global best position); $rand_1$ and $rand_2$ are the independent uniformly random 95 numbers in the range [0,1]; c_1 and c_2 are the positive learning factor which controls the 96 maximum step length; w is the inertial weight factor that controls the speed of the particles. Equation (7) gives the updated velocity based on the current velocity, current position, local, 97 98 best position and global best position. This process is repeated until the desired result is 99 obtained. The schematic diagram/flow chart of PSO algorithm is shown in Figure 2.

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101 Examples

102 The Matlab code based on PSO is applied to a synthetic model of a sedimentary basin and

103 real field data sets from Gediz Graben, Western Anatolia and Godavari sub-basin, India.

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105 Synthetic Example

We have used a synthetic gravity anomaly of 45×10^3 m length at 1×10^3 m station interval. 106 In Bott's algorithm, the prism will be of equal width of 1×10^3 m but with different strike 107 lengths. Here parabolic density function is used with the constants $\Delta \rho_{0} = -0.65 \times 10^{3} \text{ Kg} / \text{m}^{3}$ 108 and $\alpha = 0.04 \times 10^3$ Kg / m³ per 1000 m. The profile does not bisect the strike lengths of prism 109 110 and so offset distance of the profile from the centre of each prism is mentioned in the code. 111 We have added an interference term, Ax+B, with A = -0.007 mgal per 1000 m and B = -10mgal for the regional background. Required result is found at 15 iterations with RMS error of 112 113 2.9369e-006 from Marguardt algorithm.





114 We have used the same synthetic gravity anomaly for PSO algorithm. The Figure 3 115 shows the learning process of Pbest and G Best in terms of error and iterations. The best 116 result is found with 57 iterations and 50 numbers of models (Figure 3). So it is seen that after 117 57 iterations and 50 models, the calculated anomalies match the synthetic anomaly and 118 estimated depths coincide with the actual structure where RMS Error is 2.8383e-004. 119 Gaussian noises of 5% and 10% are added to the synthetic data to perceive the robustness of 120 the PSO algorithm. PSO does not find the true depths, but give values close to the true depths. The upper part of Figure 4 shows the synthetic and PSO calculated gravity anomalies 121 122 of a synthetic model of a 2.5-D sedimentary basin and the lower part shows the inferred depth 123 structure obtained from PSO and Marquardt algorithm for synthetic data. Figure 5 and 124 Figure 6 shows the synthetic data with 5% and 10% Gaussian noises and calculated gravity 125 anomalies obtained from PSO algorithm and inferred depth structure obtained by PSO and 126 Marquardt algorithm.

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128 Field Example

129 Gediz Graben, Western Anatolia

The first field case study of the interpretation of gravity anomalies has been taken from Gediz Graben, Western Anatolia. The PSO technique has been applied using 29 vertical prisms, each with equal width of 2×10^3 m but with different strike length, whereas Chakravarthi and Sundararajan (2007) used same prism and interpreted gravity anomaly by Marquardt algorithm using a parabolic density function whose constants are $\Delta \rho_0 = -1.407 \times 10^3$ Kg / m³ and $\alpha = 2.26935 \times 10^3$ Kg / m³ per 1000 m...

We have used a similar number of prisms in PSO to improve the results. So with 65 iterations and 50 models, we achieve a good fit between observed and PSO analyzed gravity





- anomalies with RMS error of 0.0083. The maximum thickness of the graben is inferred as 139 1.87×10^3 m that matches well with 1.8×10^3 m as estimated by Sari and Salk (2002) as 140 compared to 1.64×10^3 m obtained by Chakravarthi and Sundararajan (2007). The upper part 141 of Figure 7 shows the observed and PSO calculated gravity anomalies over Gediz Graben, 142 Western Anatolia and the lower part show the inferred depth structure obtained from PSO 143 and Marquardt algorithm.
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145 Godavari sub-basin

146 The Godavari sub-basin is one of the major basins of Pranhita-Godavari valleys (Rao, 1982), 147 whose approximate strike length is 220×10^3 m. The gravity profile is taken for study from the residual bouguer gravity anomaly map of the Godavari sub-basin as shown in Figure 8. 148 We have used 29 vertical prisms, each with equal widths of 2×10^3 m but with different 149 150 strike length for sedimentary basin modeling. The constants of parabolic density functions 151 used for Godavari sub-basin are $\Delta \rho_{a} = -0.5 \times 10^{3} \text{Kg} / \text{m}^{3}$ and $\alpha = 0.1518259 \times 10^{3} \text{Kg} / \text{m}^{3}$ 152 per 1000 m. (Chakravarthi and Sundararajan, 2004). So with 71 iterations and 45 models, we 153 achieve a good fit between observed and PSO analyzed gravity anomalies. The RMS error is 154 0.0099. The maximum depth of the basin, obtained from PSO is 4.09×10^3 m which is quite 155 close to the borehole information (Agarwal, 1995). Chakravarthi and Sundararajan (2005) obtained maximum depth of 4.0×10^3 m whereas Ramanamurty and Parthasarathy (1988) 156 suggested 4.5×10^3 m as the thickness of the basin. The upper part of Figure 9 shows the 157 158 variation of observed and PSO calculated gravity anomalies of Godavari sub-basin and the 159 lower part shows the inferred structure obtained from PSO and Marquardt algorithm.

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162 Conclusions

163 Particle swarm optimization (PSO) on Matlab environment is developed to estimate the 164 model parameters of a 2.5-D sedimentary basin where the density contrast varies 165 parabolically with depth. We have implemented the PSO algorithm on synthetic data with 166 and without Gaussian noise and two field data sets. An observation has made that PSO is 167 affected by some levels of noise, but estimated depths are close to the true depths. The results 168 obtained from PSO using synthetic and field gravity anomalies are well correlated with borehole 169 samples and provide more geological viable than Marquardt results. Despite its long computation 170 time, PSO is very simple to implement and is controlled by only one operator i.e. velocity 171 updating.

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266	Figure 2. The detail schematic diagram/flow chart of PSO techniques.
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268	synthetic gravity anomaly.
269	Figure 4. Synthetic and Calculated gravity anomalies with parabolic density function due to a
270	synthetic model of a 2.5-D sedimentary basin, obtained from PSO algorithm and
271	inferred depth structure obtained from PSO and Marquardt algorithm.
272	Figure 5. Synthetic data with 5% Gaussian noise and calculated gravity anomalies obtained
273	from PSO algorithm and inferred depth structure obtained from PSO and Marquardt
274	algorithm.
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276	from PSO algorithm and inferred depth structure obtained for PSO algorithm and
277	Marquardt algorithm.
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279	Graben, Western Anatolia obtained from PSO algorithm and Inferred structure
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281	Figure 8. Residual bouguer gravity anomaly map of Godavari sub-basin (modified after
282	Chakravarthi and Sundararajan, 2005) and gravity anomaly profile taken for study.





283	Figure 9. Observed and calculated residual bouguer gravity anomalies of parabolic density
284	function of Godavari sub-basin obtained from PSO algorithm and inferred depth
285	structure from PSO and Marquardt algorithm.
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Geoscientific Instrumentation Methods and Data Systems Discussions





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