1	Geological Stratigraphy and Spatial Distribution of
2	Microfractures over Costa Rica Convergent Margin, Central
3	America- A Wavelet-Fractal Analysis
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7	Abstract
8	Identification of spatial variation of lithology as a function of position and scale is very critical
9	job for lithology modeling in industry. Wavelet Transform (WT) is an efficacious and powerful
10	mathematical tool for time (position) and frequency (scale) localization. It has numerous
11	advantages over Fourier Transform (FT) to obtain frequency and time information of a signal.
12	Initially Continuous Wavelet Transform (CWT) is applied on gamma ray logs of two different
13	Well sites (Well-1039 & Well-1043) of Costa Rica Convergent Margin, Central America for
14	identifications of lithofacies distribution and fracture zone later Discrete Wavelet Transform
15	(DWT) applied to DPHI log signals to show its efficiency in discriminating small changes
16	along the rock matrix irrespective of the instantaneous magnitude to represent the fracture
17	contribution from the total porosity recorded. Further the data of the appropriate depths
18	partitioned using above mathematical tools are utilized separately for wavelet based fractal
19	analysis (WBFA). As consequences of CWT operation it is found that there are four major
20	sedimentary layers terminated with a concordant igneous intrusion passing through both the
21	wells. In addition of WBFA analysis clearly understand the fractal dimension value is
22	persistent in first sedimentary layers and the last gabbroic sill intrusions Inconsistent value of
23	fractal dimension is attributed to fracture dominant in intermediate sedimentary layers it is also

validate through core analysis. Fractal Dimension values suggest that the sedimentary
 environments persisting in that well locations bears abundant shale content and of low energy
 environments.

- 27 **Key words**: CWT, DWT, Fractal, Costa Rica.
- 28

29 Introduction

The nature of any log signal is fluctuating type in accordance to the subsurface geology. A 30 gamma ray log is most vividly used log for lithology identifications. These signals are very 31 32 noisy in some cases and highly fluctuating in another way. Manual interpretations of such signals are quite difficult and it needs more experience. These difficulties are minimized by 33 kind of wavelet transform method. In our study Continuous Wavelet transform (CWT) is tested 34 on generated synthetic signals and applied to field data. The analyzed results prove that the 35 CWT is highly suitable in geophysical log signals whereas conventional Fast Fourier 36 Transform (FFT) fails in this case because it considers the whole signal in a stationary form. 37 Though WT provides unambiguous results in analyzing the noisy and non-stationary signals, 38 its efficiency of extracting the information from the signal was seen through its wavelet 39 40 coefficients (Hui and Zaixing, 2010) with wavelet scalogram.

Number of publication has come to identify the lithofacies boundary using various mother
Wavelet transform and Fourier transform (Chandrasekhar et al., 2012; Coconi et al., 2010;
Dashtian, 2011; Javid and Tokhmechi, Mansinha et al., 1997; Mansinha 2003, 2004; Pan et
al., 2007, 2012; Pinnegar and Stockwell, 2007; Stockwell et al., 1996; Sahimi and Hashemi,
2001; Tokhmechi et al., 2009a, b; Yue et al., 2004; Zhang et al., 2011). Some other authors
worked on WT to describe the scaling property using magnetic susceptibility data (Fedi, 2003)

and Bansal et al., (2010) has determine the presence of fractures using power law scaling
behavior of magnetic susceptibility and density variation in continental crust.

In this paper, CWT and Discrete wavelet transform (DWT) are used separately for 49 identifying the lithology using gamma ray log data of well site 1039 and 1043 obtained from 50 Costa Rica Convergent Margin, Central America (expedition 308 scientists 2005) and 51 computed wavelet scalograms. Moreover, the information of fractures zones is analyzed with 52 DWT using density logs data for both wells that provides well featured whereas the log data 53 doesn't carry information of fracture remains featureless. Afterward, a linear relationship is 54 obtained between the fracture density obtained through DWT and identified fractures from 55 water saturation logs using above methods. Apart from wavelet analysis, one of the approach 56 analysis (WBFA) techniques wavelet based fractal applied to attribute 57 the roughness/smoothness of the fractures. The obtained suggest that wavelet transform acts as a 58 microscope to delineate the high and low frequency hidden in the signal separately, 59 wavelet/holder exponent and fractal dimension are highly useful in identification of lithofacies 60 and spatial distribution of fractures. 61

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63 Mathematical Background

64 Wavelet Transform

Wavelet transform is mathematical tool that can be used to analyze both stationary and nonstationary signals (Daubechies 1990, 1992) and expand time series into time frequency space. Therefore, this method can find localized intermittent periodicities. For analyzing stationary or non-stationary signal proper mother wavelet has to be substituted and the operation of continuous wavelet transform (CWT) proceeds as the convolution between time series of our

70 interest. The Discrete wavelet transform (DWT) is very useful in case of noisy data it compresses the data by reducing noise and improves the resolution whereas the application of 71 CWT prefers to extract the lithological feature from data. As it exposes the signal to high and 72 73 low frequency filters to form approximate and detailed coefficients traces out the abrupt changes in the signal. Basically, in geophysical well logs the abrupt change corresponds to its 74 own individual parameter changes which provide us more information about the subsurface 75 stratigraphy. This methodology pertaining to DWT allows us to locate the high frequency 76 changes immersed in the log which cannot be identified manually. For example, gamma ray 77 78 log is a good lithology indicator but in certain conditions it is highly fluctuating in nature. This nature sometimes perturbs its evaluation. Apart from lithology identification, DWT provides 79 an advantage of analyzing the fracture identifications. Choice of mother wavelet is important 80 factor for analyzing the non- stationary signal. As number of mother wavelets were tested to 81 select the optimum wavelet using well logs signal (Lopez 2006). In this method analysis is 82 based on linearity between logarithm of wavelet coefficients (log σ) and scale. Regression 83 Coefficients R² for all log signal from each well has been calculated and linear fit was obtained 84 for Coiflet 4 wavelet. The wavelets do not give the significant change in wavelet coefficients 85 to identify stratigraphy boundary. Thus Coiflet 4 wavelet is best for analysis of this well log 86 data. 87

- 88
- 89 **2.2 Continuous Wavelet Transform**

90 The concept of continuous wavelet transform can be explained by a basic equation given91 below:

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$$W(a,b) = \frac{1}{a^n} \int_{-\infty}^{\infty} f(x)\varphi\left(\frac{x-b}{a}\right) dx$$
(1)

93 Where, f(t) is the time series of our interest, $\varphi(t)$ is the mother wavelet, a is the scaling 94 parameter which is inverse of frequency, b is the translation parameter directly proportional to 95 time and n is the normalizing parameter which is equal to 1 (say). The variance of Wavelet 96 coefficients follows power law relation with the scale which can be given by a simple equation 97 given below;

98

$$v = x^n$$

Here v is the variance of wavelet coefficients, x is the scale and h is the holder/waveletexponent.

Holder/Wavelet exponent provides the measure of roughness/smoothness. If the holder exponent values are high, it accounts for smoothness whereas low values of holder exponent emphasis more roughness. After obtaining the holder exponent it can be substituted in the equation given below to obtain the fractal dimension value;

105
$$2D = 5 - h$$

Where D is the fractal dimension (FD) that is computed using the holder exponent and varianceof Wavelet coefficient known as Wavelet based fractal analysis (WBFA).

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109 **2.3 Discrete Wavelet Transform**

110 One- dimensional Discrete Wavelet Transform has been carried out in this task as per the 111 datasets, which are discrete and one dimensional. For the construction of DWT one sets, $a = 2^{j}$ 112 and $b = 2^{j}k$, where j and k are both integers. 1-D DWT is given by the following equation,

113
$$D_j(k) = 2^{-\frac{j}{2}} \int_{-\infty}^{\infty} f(t) \,\varphi \Big(2^{-j} t - k \Big) dt \tag{2}$$

114	Where $f(t)$ is the time series of our interest and $k = 1, 2, 3, n$ where n being the discrete
115	data array of maximum size. Time series data of our interest is decomposed to approximate
116	and detailed coefficients providing both lower and higher frequency information respectively.

118 **3.0 Results and Discussions**

3.1 Application to Synthetic data

A Synthetic signal is generated with three different frequencies such as 3Hz, 5Hz and 10Hz 120 121 and analyzed by CWT and also applied to synthetic signal added with 25% Gaussian white noise. The result obtained is shown in Figure 1. As the signal is free from noise possessing 122 only its own frequencies the mathematical tools didn't posed any difficulty and the information 123 required are derived without any ambiguity. When the same signal analyzed by the above 124 mentioned techniques after mixing noise, it provides large difference in the results which are 125 shown in Figure 2. The CWT provides an acceptable picture in analyzing the non-stationary 126 127 as well as the same non-stationary signal mixed with noise. CWT not only removes the ambiguity through by forming wavelet modulus maxima but also through its Wavelet 128 Coefficients. Also it provides a picture of the Time-Frequency localization in interpretable 129 130 form. An advantage pertaining to wavelet transform is that the Wavelet coefficients records the exact information of the signal even it is noisy. This notion regarding CWT proves it as a 131 132 good tool for identification of lithology in Well logs. Therefore, this technique can be used in 133 all circumstances to derive the exact information in the signal.

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Mostly, Porosity logs are used for this approach and the fluctuating nature of the porosity logscan be correlated to both pores distribution and the fracture (major as well as several micro

137 fractures) as well. DWT differentiates both fractures and the characteristics of the pores in the 138 detailed coefficients (Sahimi and Hashemi, 2001). For demonstration of the techniques, we have generated two type of synthetic well logs (i) assuming well site is fractureless and (ii) 139 140 well site is fractured. Now wavelet detail coefficient (WDC) for both well site are calculated as sown in in Figure 3(a & b) and Figure 3(c & d). We observed from WDC analysis that will 141 be containing highly differentiable features in terms of spikes or local maxima as shown in 142 Figure 3(d). The noisy data points pertaining to the uniform distribution constitute both low 143 and high values in comparison with its surrounding data points. DWT differentiates these 144 particular locations by means of a spike irrespective of the magnitude of the data points 145 replaced. DWT exposes the signal to low and high frequency filters produces detailed and 146 approximate coefficients respectively. 147

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149 **3.2** Application of Field Data: Costa Rica Convergent Margin, Central America

Costa Rica Convergent Margin in Central America is due to the convergence of Cocos and 150 151 Caribbean Plates. A seismic migrated section over the region is shown in the Figure 4 showing Well sites 1039, 1040 and 1043. Among these wells sites 1039 and 1043 are taken for study 152 153 whereas the site 1040 is omitted as it is not passing through certain major litho-units. Logs such as gamma ray and density are taken for study and the gamma ray signals exhibiting sharp 154 spikes which are attributed to presence of interbeded ash layers. From the gamma ray log 155 156 various lithology are identified and correlated with site adjacent to it. Density Logs are used for identification of spatial distribution of fractures along the rock matrix using DWT. Also 157 the analysis of fractal dimension value through WBFA indicates the presence of fracture in 158 159 lithology. Core Analysis reports the presence of four sedimentary layers terminated by a 160 concordant Igneous Intrusion Gabbroic Sill. Well site 1039 is taken as the reference and 161 lithology identified through Wavelet Transform are correlated to the site-1043 and the result confirms the subduction zone. As conventional technique such as Fast Fourier Transform fails 162 163 in providing the time-frequency localization. So the application of wavelet Transform is the only way to find the proper time-frequency localization. The results obtained from CWT 164 analyzed using log data sets prove the lithological successions. This result is significant in 165 certain scale range only. Since scale is inverse of frequency thus small scale and high scale 166 shows high frequency component and low frequency component of signal. Wavelet analysis 167 of signal at small scale shows the very small changes in data which may be associated with 168 noise also while large scale shows the outspread view of signal. The multiscale analysis has 169 important role in computation of wavelet coefficients (Dimri et al., 2005). The scale is linear 170 171 in a particular range is determined by log(var(cofficients)) versus log(scale) as shown in Figure 5. 172

The stratigraphic interfaces occurring in the Well log-1039 (Figure 6) appears in the Well 173 174 log-1043 (Figure 7) after having disruptions in the middle. From the seismic section it is seen that there are four major lithology running from the Well -1039 to Well - 1043 and terminated 175 176 as Gabbroic Sill. The Well-1040 crossed the above mentioned strata very mildly and it didn't reach the concordant intrusive structure as reached by the Wells-1039 and 1043. Therefore, for 177 interpretation point of view only the Wells-1039 and 1043 are used. The major successions 178 179 mentioned after drilling is that the four sedimentary interfaces followed by a Gabbroic sill. The sedimentary succession obtained underneath the reference site-1039 situated in the Cocos Plate 180 found to occur in the site-1043 without any disruption. It is also noted from the observation 181 182 made by Eric et al., (2000) as the Cocos Plate subducting under the Caribbean Plate the off 183 scarping of the sediments in the Cocos Plate should occur on the overriding plate but on analyzing the chemical composition it was mentioned the sediment lying on the overriding 184 plate was of different composition. This analysis comes in support of the effort of framing the 185 subducting system of Costa Rica using CWT spectrum, it is observed that the sedimentary 186 succession in the site-1039 over the subducted Cocos Plate continuing through the site-1043 187 without any disruption situated over the overriding Caribbean plate. In accordance to the 188 locations of the Wells and the continuity of the sedimentary successions existing in the both 189 sites (1039 and 1043) as traced by correlation of Wavelet scalogram (Figure 8) where 190 191 Figure 8 suggests that the Cocos plate is being subducted under the Caribbean plate. Application of DWT applied to porosity log of both the well 1039 & well 1043 to identify the 192 presence of fracture in lithology figure 9 (a & b). 193

The lithology identified through time and frequency localization tools are used for the 194 WBFA by taking their data points separately. Table 1 shows the FD values of various 195 lithofacies over both wells. The FD values are computed these varies from 1.21 to 1.49 in the 196 197 well-1039 and 1.20 to 1.44 in the well-1043 and associated coefficient of determination, R^2 (%) are also calculated for both wells. We observed the FD and coefficient of determination 198 199 over both wells and found that there are transitional changes between sandy shale and shaly sandstone due to variation in FD values and this variation corresponds to a gradual transition 200 between different sedimentary environments. Hence, the FD values can be used as a well log 201 attribute. 202

Here the FD values are greater than 1.2 that emphasize the presence of high shale content and low energy environment in depth range 210 to 330m and 315 to 430m as reported by (Lopez 2006) in the presence of sandstone over the well sites 1039 and 1043 respectively (Figure 10 &11). In spite of the presence of sandstone, the FD values are exceeding 1.2
indicating the dominance of shale content and these values are found to be not consistent from
reference and 1043 site. In prior depth ranges, the inconsistency of FD values are attributed to
the presence of fractures from the structural observations obtained in well site but in the above
mentioned depth ranges the inconsistency as well as from the holder exponent values it is noted
that the roughness exists in the particular lithology. The analyzed results are well correlated
with the core samples.

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214 **7.** Conclusions

Lithology identification is a tedious job in well logging and it is the most important one for reservoir characterisation. To identify Presence of structural feature such as fracture by quick look interpretation methods is very difficult using well log data. Formation micro imager (FMI) log often used to identify it is very expensive. Thus methodology used for lithology and fracture identification using wavelet transform and wavelet based fractal analysis using holder exponent can be a useful stuff to extract the different lithological feature as well as stratigraphy feature.

For structural feature identification from various lithology holder exponent and fractal dimension values can be utilised and in the presence of some extra information as that of the structural observations from well sites the results can be more promising. In order to avoid the assistance of extra information more datasets are needed from the same area so that on application of WBFA on various lithologies passing through the area provides concrete idea on lithology and Structural features using holder exponent and fractal dimension values.

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231	comments to improve our manuscript.
232	
233	Figure and Table Captions
234	Figure 1: Shows the Continuous Wavelet Transform (CWT) using a synthetic time series data.
235	Figure 2: Shows the Continuous Wavelet Transform (CWT) of a synthetic noisy time series
236	data.
237	Figure 3: (a) synthetic well logs data over the fractureless well site, (b) Discrete wavelet detail
238	coefficient (DWC) of fractureless well site, (c) synthetic well log data over the
239	fractured well site, and (d) Discrete wavelet detail coefficient (DWC) of fracture less
240	well site.
241	Figure 4: Shows the seismic migrated section showing the Wells (after Erik et al, 2000)
242	Figure 5: shows the scale of interest shows variance of wavelet coefficients versus scale of
243	gamma ray of well site 1039 and 1043.
244	Figures 6: showing Continuous Wavelet Transform (CWT) using gamma ray signal and the
245	Wavelet Coefficient at an altitude-32 of the gamma ray log of the Well location-1039
246	Figure 7: showing Continuous Wavelet Transform (CWT) using gamma ray signal and the
247	Wavelet Coefficient at an altitude-32 of the gamma ray log of the Well location-1043
248	Figure 8: Represents the lithology identification using the gamma ray log of the Well site 1039
249	and 1043 by the lines drawn on the scalogram and it represents the subduction zone in
250	the areas obtained from the seismic migrated section.

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251	Figure 9: (a) Shows the discrete detailed and approximate coefficients and the spikes represents
252	the possible fractures at well location 1039, (b) shows the Discrete detailed and
253	Approximate coefficients and the spikes represents the possible fractures at well
254	location 1043
255	Figure 10: Shows variance of Wavelet coefficients versus scale of density log of well site-1039
256	and 1043 which shows consistent holder exponent and fractal dimension values
257	indicating that wells contains similar sedimentary environment.
258	Figure 11: Shows the FD values of both well sites of 1039 and 1043.
259	Table 1: Shows the FD values of the appropriate lithology identified and the circled depth
260	ranging and its appropriate fractal dimension values showing deviation of the vales
261	from the reference site 1039.
262	Table 2: Shows the ranges of fractal dimension values.
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341 Figure 1







387 Figure 3























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571 Figure 11



594 Table 1

596		Depth Range (m.)		Fractal Dimension		Coefficient of	
	Lithofacies					determinat	tion, R^2 (%)
597		Well 1039	Well 1043	Well 1039	Well 1043	Well 1039	Well 1043
598	Shale with	20-80	60 - 130	1.21	1.22	99.441	99.5988
	interbeded ash						
599	layer						
	Shaly sandstone	80 -160	130 - 260	1.36	1.43	99.3234	99.3375
600							
601	Sandy shale	160 - 210	260-315	1.26	1.44	99.0514	98.8141
601	with interbeded						
602	ash layer						
002	Sandstone	210 - 330	315-430	1.49	1.39	98.8141	98.791
603							
003	Gabbroic sill	330 - 400	430-450	1.20	1.20	99.1356	96.96441
604							
004							

617 Table 2

619	Fractal Dimension	Interpretation
620	< 0.9	High sand content and high energy environment
621	0.9 to 1.2	Inter-bedded sand and shale
(22	> 1.2	High shale content and low energy environment
622		