



Inversion of Residual Gravity Anomalies using Tuned-PSO Technique

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6 Abstract

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Many kinds of particle swarm optimization (PSO) technique are now available and various 7 8 efforts have been made to solve linear and non linear problems as well as one dimensional 9 and multidimensional problem of geophysical data. Particle swarm optimization is a Meta heuristic optimization method that requires the intelligent guess and suitable selection of 10 11 controlling parameters (i.e. Inertia weight and acceleration coefficient) for better convergence at global minima. The proposed technique Tuned-PSO is an improved technique of PSO, in 12 13 which effort has been made for choosing the controlling parameters and these parameters have selected after analysing the response of various possible exercises using synthetic 14 gravity anomalies over various geological sources. The applicability and efficacy of the 15 proposed method is tested and also validated using synthetic gravity anomalies over various 16 source geometries. Finally Tuned-PSO is applied over field residual gravity anomalies of two 17 18 different geological terrains to find out the model parameters namely amplitude coefficient factor (A), shape factor (q) and depth (z). The analysed results have been compared with 19 published results obtained by different methods that show a significantly excellent agreement 20 with real model parameters. The results also show that the proposed approach is not only 21 superior to the other methods but also shows that the strategy has enhanced the exploration 22 capability of proposed method. Thus Tuned-PSO is an efficient and more robust technique to 23 24 achieve optimal solution with minimal error.

25 Keywords: Tuned–PSO, gravity anomalies, inversion.

26

27 **1. Introduction**

Gravity method is based on the measurement of gravity anomalies caused by the density variation due to source anomalies. Gravity method has been used in a wide range of application as a reconnaissance method for oil exploration and as a secondary method for mineral exploration, to find out the approximate geometry of the source anomalies, bedrock depths, and shapes of the earth. Interpretation of geophysical data that involves solving an

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inverse problem; many techniques have been developed to invert the geophysical data to 33 34 estimate the model parameters. These methods can be broadly categorised into two groups: (1) local search technique (e.g. Steepest descent method; conjugate gradient method, ridge 35 regression, Levenberg- Marquardt method etc.) and (2) global search techniques (e.g., 36 simulated annealing, genetic algorithms, particle swarm optimization, Ant colony 37 optimization etc.) Local search technique is simple and requires a very good initial 38 presumption - close to true model for a successful convergence. In other hand global search 39 40 method may provide an acceptable solution but computationally time intensive. There are 41 several local and global inversion technique has been developed to interpret gravity anomalies (Thanassoulas et al., 1987; Shamsipour et al., 2012; Montesinos et al., 2005; Qiu, 42 2009; Toushmalani, 2013). However, PSO has been successfully applied in many fields, such 43 as model construction, biomedical images, electromagnetic optimization, hydrological 44 problem etc. (Cedeno and Agrafiotis, 2003; Wachowiak et al., 2004; Boeringer and Werner, 45 2004; Kumar and Reddy, 2007; Eberhart and Shi, 2001; El-Kaliouby and Al-Garni, 2009) but 46 in the geophysical field PSO has limited number of applications (Alvarez et al., 2006; Shaw 47 48 and Srivastava, 2007).

In this paper improved Particle Swarm Optimization known as Tuned-PSO with fine tuning of learning parameters have been tested using synthetic gravity anomalies over kinds of geometrical bodies and compared their efficacy. On the basis of performance, finally Tuned PSO has been used to invert gravity anomalies to find out the essential model parameters such as shape factor (q), depth (z), amplitude coefficient factor (A) and horizontal location of the source geometry.

55 2. Forward modelling for generating the synthetic gravity anomalies

A general expression of gravity anomaly caused by a sphere, an infinite long horizontal cylinder and a semi-infinite vertical cylinder have been used for generating the gravity anomalies in forward problem that is given in equation 1 (Abdelrahman *et al.*, 1989) as follows:

$$g(\mathbf{x}_{i,Z},\mathbf{q}) = \mathbf{A} \frac{z^m}{(\mathbf{X}^i + \mathbf{Z}^2)^{\mathbf{q}}}$$
(1)

61 Where

60

62
63
$$A = \begin{cases} \frac{4}{3}\pi G\sigma R^3 \text{ for a sphere }, \\ 2\pi G\sigma R^2 \text{ for a horizontal cylinder,} \\ \pi G\sigma R^2 \text{ for a vertical cylinder,} \end{cases} m = \begin{cases} 1, \\ 1, \\ 0, \end{cases}$$

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what is basic difference between Tuned PSO and PSO. Author can discuss the demerit of PSO technique and possible compare the present result with PSO.



(3)



65 66 67 $q = \begin{cases} \frac{3}{2} & \text{for sphere,} \\ 1 & \text{for horizontal cylinder,} \\ \frac{1}{2} & \text{for a vertical cylinder ; R<<Z.} \end{cases}$

69

70 Where A, q and z represent amplitude coefficient factor, shape factor and depth respectively; 71 and x_i , σ , G and R are the position coordinate, density contrast, universal gravitational constant and radius of geometrical bodies respectively. 1pr semi-infinite vertical cylinder the 72 gravity response is only applicable when the radius of the cylinder is much smaller than the 73 distance from observation position to the top of the cylinder. In the forward modelling for 74 generating the synthetic gravity anomalies, the 3-private coefficient factor of 600 75 mGal*km² and 200 mGal for sphere and vertical cylinder respectively, correspond to the 76 shape factor as 1.5 and 0.5, and the depth of 5.0 km and 3.0 km are used. The shape factor 77 approaches to zero as the structure becomes a nearly horizontal bed and approaches 1.5 as the 78 structure becomes a perfect sphere (point mass). As in the formulae x_i is the position 79 coordinate; at the origin $x_i = 0$ then equation 1 becomes, 80

81
$$g(0) = \frac{A}{z^{2q-m}}$$
 (2)

82 The equation 3 is taken for addition of 10% white Gaussian noise.

$$g_{noisy}(x) = awgn(g(x), 0.1)$$

84

85 3. Tuned- Particle Swarm Optimization (Tuned-PSO)

Tuned-Particle Swarm Optimization (Tuned-PSO) is an improved Particle swarm 86 87 optimization (PSO) method after the fine tuning of its learning parameters. The concept of 88 PSO is described as follows (Eberhart and Kennedy, 1995): (a) each potential solution called as particles and knows its best values so far (P_{best}) and its position more over each particle 89 knows the best value in the group (G_{best}) among the P_{best} . All of the best values are based on 90 objective function (Q) for each problem to be solved. Each particle tries to modify its position 91 92 through the current velocity and its positions. The velocity of each particle can be updated 93 using the following equations (Santos, 2010):

94
$$v_{i}^{k+1} = \omega^{k} v_{i}^{k} + c_{1} rand() * (P_{best-i}^{k} - x_{i}^{k+1}) + c_{2} rand() * (G_{best}^{k} - x_{i}^{k+1})$$
95
$$x_{i}^{k+1} = x_{i}^{k} + v_{i}^{k+1}$$
(4)

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it is universal	or its assumed for computation in p	resent studies.
Pl mention th	ne reference.	

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96 Where v_l^k is the velocity of i_{th} particle at k_{th} iteration, x_l^k represents current position of i_{th} 97 particle at k_{th} iteration, *rand()* is a random number in the range of 0 and 1. $c_1 \& c_2$ are 98 constants known as cognitive coefficient and social coefficient respectively. The coefficient 99 c_1 has contribution towards the self exploration of a particle and the coefficient c_2 has a 100 contribution towards the motion of the particles in global direction, and ω is an inertia 101 weight in the range [0, 1]. The objective function has calculated by following equation 102 (Santos, 2010).

103
$$Q = \frac{2\sum_{i}^{N} |v_{i}^{o} - v_{i}^{c}|}{\sum_{i}^{N} |v_{i}^{o} - v_{i}^{c}| + \sum_{i}^{N} |v_{i}^{o} + v_{i}^{c}|}$$
(5)

Where N is the number of iteration, v_i^o and v_i^c are observed and calculated gravity anomaly
 measured at point p(x_i) respectively.

106

107 4 Discussion and Results

108 4.1 Selection of learning parameter for Tuned-PSO Modelling

In this paper, a judicious selection of the parameters (i.e. ω , c_1 , and c_2 ,) has been discussed for controlling the convergence behaviours of Tuned-PSO based algorithm. The settings of these parameters determine how it optimizes the search-space. These algorithms with suitable selection of parameter become more powerful global search algorithm for their practical applications.

114 4.1.1 Inertia weight

Inertia weight ω controls the momentum of the particle (Eberhart and Shi, 2001; Eberhart and Kennedy, 1995). Here two kinds of source geometry are adopted to evaluate more suitable ranges of parameters in the Tuned-PSO. Tuning of inertia weight, 0, 0.4, 0.7, 0.9, has been taken for two different acceleration coefficients at 1.4 and 2.0 respectively. From Figure 1, it is clear that the best convergence has performed by algorithm at inertia weight 0.7. This value of inertia weight produces high convergence rate at less number of iteration than the other values.

122 4.1.2 The maximum velocity v_{max}

123 The maximum velocity v_{max} determines the maximum change one particle can undergo in its

124 positional coordinates during iteration and used to avoid explosion and divergence. Usually,

125 the full search ranges of the particle's positions as the vmax are fixed. For example, in case, a

particle has position vector $x = (x_1, x_2, x_3)$ and if $-15 \le x_i \le 15$ for i=1, 2 and 3, then $v_{max} =$

127 30 is fixed.

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129 4.1.3 The swarm size

It is quite a common practice in the PSO literature to limit the range of number of particles. Van den Bergh and Engelbrecht have shown that though there is a slight improvement of the optimal value with increasing swarm size, a larger swarm increases the number of function evaluations to converge to an error limit. However, Eberhart and Shi Illustrated that the population size has hardly any effect on the performance of the PSO method. So, in this paper population size has taken 100.

136

137 **4.1.4** The acceleration coefficients c_1 and c_2

To find the best tuning of learning parameters, various values of c_1 , c_2 (i.e. $c_1 = c_2 = 1.0$, 1.2, 1.4, 1.6, 1.8, and 2.0) and inertia weights (i.e. 0.4, 0.7 and 0.9) are taken, and various exercises have been made using the two different geometrical bodies by fixing the each of the inertia weight (Table 1). The results was analysed and found that more suitable values of c_1 and c_2 (i.e. $c_1 = c_2 = 1.4$) are the best tuned acceleration coefficients for our case. These values of acceleration coefficients have been used to invert the gravity anomalies, which provide significant improvement and produce optimal solutions of the geological bodies.

145

146 **4.2** Application to Synthetic gravity anomalies

Initially two geometrical models i.e. sphere and vertical cylinder has been considered for 147 testing the applicability and efficacy of Tuned-PSO. The efficacy of proposed algorithm in 148 terms of RMS error versus iterations is as shown in Figure 1. The gravity anomalies over 149 150 these models are computed from equation (1) for the model parameters as shown in Table 1(a, b) and 2(a, b). In each case, the length of gravity profile of 51 km has 51data points at 151 one km equal interval. 2 gravity anomaly for every source model is corrupted with 10% of 152 white gaussian noise and Tuned-PSO based inversion algorithms applied on them. The 153 optimized results obtained by Juned-PSO algorithms for synthetic noise free and with 10% 154 noisy data und 1 shows that tuned-PSO has best results at values 1.4, 1.4 and 0.7 for 155 c1, c2 and inertia weight (w) respectively. This also shows that Tuned-PSO curve is having 156 157 less number of local minima than other values. It means that the Tuned-PSO technique 158 minimise the number of local minima for solving the geophysical nonlinear inverse problems. The 5 mulated gravity anomaly by Tuned-PSO and computed gravity anomaly are shown in 159 160 Figure 2(a) and 3(a) respectively and correspond to 7 nthetic gravity anomaly and 8 mputed

161 9 lomaly corrupted with 10% of white gaussian noise as shown in Figure 2(b) and 3(b).

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Figure 2 and Figure 3 show the well matching between the 2 minetic curves and Tuned-PSO 162 calculated gravity anomalies curves over spherical model and vertical cylindrical model 163 respectively. Figure 4 shows the behaviour of pbest and gbest variation inside the algorithm and 164 ts that gbest decreases more rapidly toward the minimal error with high convergence. 165 4 e observed from Table 2 and 3 that the RMS error increases with increasing the noise in 166 gravity anomaly however, the horizontal location (x_0) is a substantially stable parameter and 167 168 varies in a small scale. 169 170 4.3 Application to Field gravity anomalies 171 4.3.1. Mobrun Sulphide Body, Near Rouyn-Noranda, Canada Mobrun polymetallic deposit near Rouyn- Noranda comprises two complexes of massive 172 173 lenses within mainly felsic volcanic rocks of the Archean Blake River Group (Barrett. et al., 1992). Gre main lens contents mainly massive sulphide, approximately 3.37 Million Ton 174 with some other elements in least amount in comparison to sulphide are 0.95 Million Ton at 175 0.81% Cu, 2.44% Zn, 30.3 g/t Ag, and 2.2 g/t Au estimated in 1989. 176 177 located at 250 m to southeast of the Main complex. Host volcanic rocks of main complex are 178 mostly massive, breciated, and tuffaceous rhyolites. Mobrun ore body is located at shallow depth; top of the body approximately 30 m depth and extended to 175 m. 179 Tuned-PSO in MATLAB environment has been applied to field residual gravity 180 anomaly. This anomaly profile of length 268 m has been taken from the Mobrun sulphide 181 body, Noranda, Canada (Nettleton, 1976; Essa, 2012). It is seen from Figure 5 that both 182 curves phalysed from Tuned-PSO and observed gravity anomalies are extremely well 183 correlated with optimal RMS error of 0.0271%. The results in terms of model parameters 184 (amplitude coefficient factor, shape factor and depth) over the Mobrun ore body analysed 185 from Tuned-PSO method can seen in Table 4(a). This table provides the optimum results 186 obtained from Tuned-PSO with 0.0271% error agrees well with the results obtained from 187 10 Iculated value of shape factor, q is 0.77 (Table 4a). This value over 188 other methods 189 Mobrun sulphide ore body reflects the shape of a semi-infinite vertical cylindrical geological 190 body is present at depth of 30 m [11] an be seen from Table (b) 12 values of amplitude 191 coefficient factor, shape factor and depth correspond to 60.0, 0.77 and 30 are more stable and consistent with results analysed from various authors 13 192 193 194 195

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196 4.3.2. Louga Anomaly West coast of Senegal, West Africa

197 The study area Louga anomaly of west coast of Senegal is taken for another case study for interpretation of gravity data using Tuned -PSO. The Senegal basin is part of the north-west 198 199 African coastal basin- a typical passive margin basin opening west to the Atlantic. The 200 complexities of the rift tectonics of the Atlantic opening gave rise to a series of sub-basins aligned north-south. The pre-rift (Upper Proterozoic to Palaeozoic), syn-rift (Permian to 201 Lower Jurassic) and post-rift are divided into a number of sub-basins, controlled by east west 202 203 transform related lineaments (Nettleton, 1962). In this paper Tuned-PSO in MATLAB 204 environment has been also applied to another field case study. Gravity anomaly of Louga area, West coast of Senegal, West Africa (Essa, 2014) has taken for Tuned-PSO analysis as 205 shown in Figure 6 has Profile length 32 km. The results in terms of model parameters 206 207 (amplitude coefficient factor, shape factor and depth) over the Louga anomaly analysed from Tuned-PSO method can seen in Table 5(a). It is seen from Figure 6 that both curves any last 208 209 from Tuned-PSO and observed gravity anomalies are extremely well correlated with optimal RMS error of 0.0271%. The optimum obtained results of model parameters amplitude 210 211 coefficient factor (A), shape factor (q) and depth (z) are 545.30 mGal, 0.53 and 4.92 km 212 respectively that shows significantly good agreement with the results obtained by various authors as shown in Table 5(b). The Tuned PSO analysed value of shape factor confirms that 213 the shape of the causative body is semi-infinite vertical cylindrical body present at depth 214 about 4.92 km. 215

216

217 5. Conclusions

In this paper, various synthetic gravity anomalies and field gravity anomalies have been 218 219 adopted for evaluating the applicability and efficacy of Tuned-PSO algorithms and also 220 determining the suitable ranges of learning parameters setting (i.e. inertia weight and acceleration coefficients). On the basis of the performance, a novel algorithm PSO with 221 suitable learning parameters has been implemented to gravity anomalies assuming models 222 223 with gravity source geometry such as sphere and vertical cylinder. This technique has been tested and demonstrated on synthetic gravity anomalies with and without gaussian noise and 224 finally applied to field residual gravity anomalies over Mobrun sulphide ore body, Noranda, 225 QC, Canada and Louga Anomaly of West coast of Senegal, West Africa. This technique 226 227 provides robust and plausible results even in the presence of noise that are consistent with the 228 results obtained from other classical methods. Thus this technique is powerful tool that

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229	improves the results of classical PSO and other technique significantly with less time and
230	optimal error.
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295	





Figure and Table Captions 296 297 Figure 1. Iteration versus RMS error plot at different acceleration coefficients and inertia 298 weights. Figure 2. (a) Synthetic gravity anomaly versus Tuned-PSO calculated gravity anomaly over 299 300 spherical model and (b) Synthetic gravity anomaly versus Tuned-PSO calculated gravity anomaly over same model with 10% white gaussian noise. 301 Figure 3. (a) Synthetic gravity anomaly versus Tuned-PSO calculated gravity anomaly over 302 303 vertical cylindrical model, (b) Synthetic gravity anomaly versus Tuned-PSO 304 calculated gravity anomaly over same model with 10% white gaussian noise. Figure 4. Iteration versus RMS error of Tuned-PSO showing pbest and gbest over synthetic 305 gravity anomaly. 306 307 Figure 5. Observed field gravity anomaly versus Tuned-PSO calculated gravity anomaly over 308 Mobrun sulphide ore body, Canada. 309 Figure 6. Observed field gravity anomaly versus Tuned-PSO calculated gravity anomaly over West Senegal anomaly, Louga area, South Africa. 310 311 Table 1. Performance of the acceleration coefficients c_1 and c_2 using the synthetic gravity 312 anomalies over spherical and vertical cylindrical geometrical bodies. Table 2. (a) Optimized model parameters, converged iteration and RMS error in the inversion 313 of synthetic gravity anomaly over a spherical source model and (b) optimized 314 parameters, converged iteration and RMS error in the inversion of synthetic gravity 315 anomaly with 10% white gaussian noise over a same source model from Tuned-PSO. 316 Table 3. (a) Optimized model parameters, converged iteration and RMS error in the inversion 317 318 of synthetic gravity anomaly over a vertical cylindrical source model and (b) optimized parameters, converged iteration and RMS error in the inversion of synthetic 319 gravity anomaly with 10% white gaussian noise over a same source model from 320 Tuned-PSO. 321 322 Table 4. (a) Analysed results and parameters (A, z and q) used to invert the gravity anomaly 323 over Mobrun sulphide ore body and (b) comparative results over Mobrun field, 324 Canada from various methods and Tuned- PSO. 325 Table 5. (a) Analysed results and parameters (A, z and q) used to invert the gravity anomaly over West Senegal anomaly, Louga area, South Africa and (b) comparative results 326 over same area from various methods and Tuned- PSO. 327 328 329





















































449 Table 1

Gravity data description	Weighting factor	$c_1 = 1.0, c_2 = 1.0$	$c_1 = 1.2, c_2 = 1.2$	$c_1 = 1.4, c_2 = 1.4$	$c_1 = 1.6, c_2 = 1.6$	$c_1 = 1.8, c_2 = 1.8$	$c_1 = 2.0, c_2 = 2.0$
				RMS	Error		
Synthetic spherical	w = 0.4	0.004899	0.002899	0.00014	0.000907	0.000853	0.000861
body	w = 0.7	0.002532	0.000118	0.000013	0.000087	0.000187	0.000247
	w = 0.9	0.005215	0.000118	0.000063	0.000379	0.000167	0.002695
Synthetic vertical	w = 0.4	0.004892	0.003231	0.000327	0.000835	0.000704	0.000932
Cylindrical body	w = 0.7	0.001913	0.000318	0.000011	0.000065	0.000207	0.000511
	w = 0.9	0.003259	0.000551	0.000189	0.000183	0.001265	0.002747





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451 Table 2

(a) Optin	(a) Optimized Parameters, converged iteration and RMS error in the inversion of synthetic								
gravity anomaly over a spherical source model.									
Z (km)	A (mGal*km ²)	q	g ₀ (mGal)	x ₀ (km)	Iteration	RMS Error (%)			
4.99883	550	1.5	24.0	-1.89x10 ⁻³	100	0.000405			
4.9999	660.31	1.5	24.0	2.39x10 ⁻⁵	200	0.000015			
5.00	610.15	1.5	24.0	-1.44x10 ⁻⁶	300	0.00			
5.00	604.36	1.5	24.0	3.3x10 ⁻¹³	400	0.00			
5.00	604.10	1.5	24.0	8.17x10 ⁻¹⁶	500	0.00			
(b) Optin gravi	nized Parameters, ty anomaly with 1	converge 0% white	ed iteration at guassian not	nd RMS error	r in the invo	ersion of synthetic e model.			
z (km)	A (mGal*km ²)	q	g ₀ (mGal)	x ₀ (km)	Iteration	RMS Error (%)			
4.5	605.49	1.5	24	-6.95X10 ⁻²	100	0.174890			
4.5	603.99	1.5	24	-6.81X10 ⁻²	200	0.174885			
4.5	550.32	1.5	24	-6.86X10 ⁻²	300	0.174883			
4.5	601.42	1.5	24	-6.86X10 ⁻²	400	0.174883			
4.5	680.0	1.5	24	-6.85X10 ⁻²	500	0.174883			

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458 Table 3

		(a) Optimized Parameters, converged iteration and RMS error in the inversion of synthetic							
gravity anomaly over a vertical cylindrical source model.									
Z (km)	A (mGal*km ²)	q	g ₀ (mGal)	x ₀ (km)	Iteration	RMS Error (%)			
3.015	182.31	0.5	66.33	-4.4×10^{-3}	100	0.001743			
3.016	185.92	0.5	66.33	-3.7x10 ⁻⁴	200	0.001635			
3.016	192.59	0.5	66.33	-2.74x10 ⁻¹⁰	300	0.001633			
3.015	162.15	0.5	66.33	-7.98x10 ⁻¹¹	400	0.001633			
3.016	169.00	0.5	66.33	-6.58x10 ⁻¹¹	500	0.001633			
(b) Optin	(b) Optimized Parameters, converged iteration and RMS error in the inversion of synthetic								
mode	ty anomaly with	10% wh	nte guassian	noise over a	vertical c	ylindrical source			
z (km)	A (mGal*km ²)	q	g ₀ (mGal)	x ₀ (km)	Iteration	RMS Error (%)			
3.02	167.33	0.5	65.81	-3.95x10 ⁻²	100	0.036732			
2.99	160.38	0.5	66.33	1.36x10 ⁻²	200	0.036968			
3.02	161.74	0.5	65.88	-4.52×10^{-2}	300	0.036672			
30.2	160.35	0.5	65.88	-4.50x10 ⁻²	400	0.036672			
3.02	198.67	0.5	65.88	-4.50x10 ⁻²	500	0.036672			

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465 Table 4

(a) Optimized Parameters, converged iteration and RMS error in the inversion of field									
gravity anomaly over Mobrun sulphide ore body.									
z (km)	A (mGal*km ²)	q	g ₀ (mGal)	$x_0 (km)$	Iteration	RMS Error (%)			
31	58.08	0.77	1.7781	-2.99078	100	0.027149			
31	59.55	0.76	1.1156	-3.02429	200	0.027163			
31	58.00	0.76	1.7826	-2.13091	300	0.027125			
31	59.03	0.77	1.7826	-2.15033	400	0.027124			
30	59.99	0.77	1.7992	-2.15013	500	0.027124			
(b) Compar	rative results over	Mobrun	field example	from vario	us methods	and GPSO.			
Parameter	Grant and West	Euler d	leconvoltuion	Fast inte	rpretation	Tuned- PSO			
	(1965)	(Roy et al., 2000) Method Method							
Z(m)	30	29.44		33	3.3	30.0			
q	-	0.77		0.	78	0.77			
A(mGal)	-		-	59	9.1	60.0			

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469 Table 5

(a) Optimized Parameters, converged iteration and RMS error in the inversion of field									
gravity anomaly over West Senegal (Louga area) anomaly.									
z (km)	A (mGal ³	*km ²)	q	g ₀ (mGal)	x_0 (km)	Iteration	RMS Error (%)		
4.90	549.4	4	0.52	94.83	-2.60x10 ⁻¹	100	0.027065		
4.90	550.	0	0.53	94.80	-2.56x10 ⁻¹	200	0.026552		
4.91	549.5	7	0.53	94.79	-2.45x10 ⁻¹	300	0.026552		
4.91	547.6	6	0.53	94.79	-2.42x10 ⁻¹	400	0.026551		
4.91	545.3	0	0.53	94.79	-2.39x10 ⁻¹	500	0.025551		
(b) Comp	arative resu	lts of v	arious n	nethods over W	Vest Senegal	(Louga area	anomaly.		
Parar	Parameter New fast least square method (Essa, 2014) Tuned-PSO method						d-PSO method		
z (km) 4.94							4.92		
0	1		0.53				0.53		
A (m	A (mGal) 545.68 545.30								