# Inversion of Residual Gravity Anomalies using Tuned-PSO Technique 

Ravi Roshan and Upendra Kumar Singh<br>Department of Applied Geophysics, Indian School of Mines, Dhanbad - 826 004, India<br>Correspondence: roshanravi.sinha@gmail.com

## Abstract

Many kinds of particle swarm optimization (PSO) technique are now available and various efforts have been made to solve linear and non linear problems as well as one dimensional and multidimensional problem of geophysical data. Particle swarm optimization is a Meta heuristic optimization method that requires the intelligent guess and suitable selection of controlling parameters (i.e. Inertia weight and acceleration coefficient) for better convergence at global minima. The proposed technique Tuned-PSO is an improved technique of PSO, in which effort has been made for choosing the controlling parameters and these parameters have selected after analysing the response of various possible exercises using synthetic gravity anomalies over various geological sources. The applicability and efficacy of the proposed method is tested and also validated using synthetic gravity anomalies over various source geometries. Finally Tuned-PSO is applied over field residual gravity anomalies of two different geological terrains to find out the model parameters namely amplitude coefficient factor (A), shape factor ( $q$ ) and depth ( z ). The analysed results have been compared with published results obtained by different methods that show a significantly excellent agreement with real model parameters. The results also show that the proposed approach is not only superior to the other methods but also shows that the strategy has enhanced the exploration capability of proposed method. Thus Tuned-PSO is an efficient and more robust technique to achieve optimal solution with minimal error.
Keywords: Tuned-PSO, gravity anomalies, inversion.

## 1. Introduction

Gravity method is based on the measurement of gravity anomalies caused by the density variation due to source anomalies. Gravity method has been used in a wide range of application as a reconnaissance method for oil exploration and as a secondary method for mineral exploration, to find out the approximate geometry of the source anomalies, bedrock depths, and shapes of the earth. Interpretation of geophysical data that involves solving an

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inverse problem; many techniques have been developed to invert the geophysical data to estimate the model parameters. These methods can be broadly categorised into two groups: (1) local search technique ( e.g. Steepest descent method; conjugate gradient method, ridge regression, Levenberg- Marquardt method etc.) and (2) global search techniques (e.g., simulated annealing, genetic algorithms, particle swarm optimization, Ant colony optimization etc.) Local search technique is simple and requires a very good initial presumption - close to true model for a successful convergence. In other hand global search method may provide an acceptable solution but computationally time intensive. There are several local and global inversion technique has been developed to interpret gravity anomalies (Thanassoulas et al., 1987; Shamsipour et al., 2012; Montesinos et al., 2005; Qiu, 2009; Toushmalani, 2013). However, PSO has been successfully applied in many fields, such as model construction, biomedical images, electromagnetic optimization, hydrological problem etc. (Cedeno and Agrafiotis, 2003; Wachowiak et al., 2004; Boeringer and Werner, 2004; Kumar and Reddy, 2007; Eberhart and Shi, 2001; El-Kaliouby and Al-Garni, 2009) but in the geophysical field PSO has limited number of applications (Alvarez et al., 2006; Shaw and Srivastava, 2007).

In this paper improved Particle Swarm Optimization known as Tuned-PSO with fine tuning of learning parameters have been tested using synthetic gravity anomalies over kinds of geometrical bodies and compared their efficacy. On the basis of performance, finally Tuned PSO has been used to invert gravity anomalies to find out the essential model parameters such as shape factor (q), depth (z), amplitude coefficient factor (A) and horizontal location of the source geometry.

## 2. Forward modelling for generating the synthetic gravity anomalies

A general expression of gravity anomaly caused by a sphere, an infinite long horizontal cylinder and a semi-infinite vertical cylinder have been used for generating the gravity anomalies in forward problem that is given in equation 1 (Abdelrahman et al., 1989) as follows:

$$
\begin{equation*}
g\left(\mathrm{x}_{\mathrm{i}, \mathrm{Z}, \mathrm{q}}\right)=\mathrm{A} \frac{\mathrm{Z}^{m}}{\left(\mathrm{Xi}^{2}+\mathrm{z}^{2}\right)^{\mathrm{q}}} \tag{1}
\end{equation*}
$$

Where

$$
\mathrm{A}=\left\{\begin{array}{l}
\frac{4}{3} \pi G \sigma R^{3} \text { for a sphere }, \\
2 \pi G \sigma R^{2} \text { for a horizontal cylinder }, \\
\pi G \sigma R^{2} \text { for a vertical cylinder }
\end{array} \quad \mathrm{m}=\left\{\begin{array}{l}
1 \\
1, \\
0
\end{array}\right.\right.
$$

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what is basic difference between Tuned PSO and PSO.
Author can discuss the demerit of PSO technique and possible compare the present result with PSO.

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$$
q= \begin{cases}\frac{3}{2} & \text { for sphere } \\ 1 & \text { for horizontal cylinder } \\ \frac{1}{2} & \text { for a vertical cylinder } ; \mathrm{R} \ll \mathrm{Z}\end{cases}
$$

Where A, q and z represent amplitude coefficient factor, shape factor and depth respectively; and $\mathrm{x}_{\mathrm{i}}, \sigma, \mathrm{G}$ and R are the position coordinate, density contrast, universal gravitational constant and radius of geometrical bodies respectively. 1 pr semi-infinite vertical cylinder the gravity response is only applicable when the radius of the cylinder is much smaller than the distance from observation position to the top of the cylinder. In the forward modelling for generating the synthetic gravity anomalies, the 2 $\mathrm{mGal}{ }^{*} \mathrm{~km}^{2}$ and 200 mGal for sphere and vertical cylinder respectively, correspond to the shape factor as 1.5 and 0.5 , and the depth of 5.0 km and 3.0 km are used. The shape factor approaches to zero as the structure becomes a nearly horizontal bed and approaches 1.5 as the structure becomes a perfect sphere (point mass). As in the formulae $x_{i}$ is the position coordinate; at the origin $\mathrm{x}_{\mathrm{i}}=0$ then equation 1 becomes,

$$
\begin{equation*}
g(0)=\frac{A}{z^{2 q-m}} \tag{2}
\end{equation*}
$$

The equation 3 is taken for addition of $10 \%$ white Gaussian noise.

$$
\begin{equation*}
g_{n o i s y}(x)=\operatorname{awgn}(g(x), 0.1) \tag{3}
\end{equation*}
$$

## 3. Tuned- Particle Swarm Optimization (Tuned-PSO)

Tuned-Particle Swarm Optimization (Tuned-PSO) is an improved Particle swarm optimization (PSO) method after the fine tuning of its learning parameters. The concept of PSO is described as follows (Eberhart and Kennedy, 1995): (a) each potential solution called as particles and knows its best values so far $\left(P_{\text {best }}\right)$ and its position more over each particle knows the best value in the group ( $G_{\text {best }}$ ) among the $P_{\text {best }}$. All of the best values are based on objective function $(Q)$ for each problem to be solved. Each particle tries to modify its position through the current velocity and its positions. The velocity of each particle can be updated using the following equations (Santos, 2010):

$$
\begin{align*}
& v_{i}^{k+1}=\omega^{k} v_{i}^{k}+c_{1} \operatorname{rand}() *\left(P_{\text {best }-i}^{k}-x_{i}^{k+1}\right)+c_{2} \operatorname{rand}() *\left(G_{\text {best }}^{k}-x_{i}^{k+1}\right) \\
& x_{i}^{k+1}=x_{i}^{k}+v_{i}^{k+1} \tag{4}
\end{align*}
$$

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| :--- |
| it is universal or its assumed for computation in present studies. |
| Pl mention the reference. |
|  |
| $=$ Number: $2 \quad$ Author: sanjay Subject: Sticky Note $\quad$ Date: 9/29/2016 11:37:46 AM | Pl check the unit of coefficient.

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Where $v_{i}^{k}$ is the velocity of $\mathrm{i}_{\text {th }}$ particle at $\mathrm{k}_{\mathrm{th}}$ iteration, $x_{i}^{k}$ represents current position of $\mathrm{i}_{\text {th }}$ particle at $k_{t h}$ iteration, $\operatorname{rand}()$ is a random number in the range of 0 and $1 . \mathrm{c}_{1} \& \mathrm{c}_{2}$ are constants known as cognitive coefficient and social coefficient respectively. The coefficient $c_{1}$ has contribution towards the self exploration of a particle and the coefficient $c_{2}$ has a contribution towards the motion of the particles in global direction, and $\omega$ is an inertia weight in the range $[0,1]$. The objective function has calculated by following equation (Santos, 2010).

$$
\begin{equation*}
Q=\frac{2 \sum_{i}^{N}\left|V_{i}^{o}-V_{i}^{c}\right|}{\sum_{i}^{N}\left|V_{i}^{o}-V_{i}^{c}\right|+\sum_{i}^{N}| |_{i}^{o}+V_{i}^{c} \mid} \tag{5}
\end{equation*}
$$

Where N is the number of iteration, $\mathrm{v}_{\mathrm{i}}{ }^{\mathrm{o}}$ and $\mathrm{v}_{\mathrm{i}}{ }^{\mathrm{c}}$ are observed and calculated gravity anomaly measured at point $\mathrm{p}\left(\mathrm{x}_{\mathrm{i}}\right)$ respectively.

## 4 Discussion and Results

### 4.1 Selection of learning parameter for Tuned-PSO Modelling

In this paper, a judicious selection of the parameters (i.e. $\omega, \mathrm{c}_{1}$, and $\mathrm{c}_{2}$, has been discussed for controlling the convergence behaviours of Tuned-PSO based algorithm. The settings of these parameters determine how it optimizes the search-space. These algorithms with suitable selection of parameter become more powerful global search algorithm for their practical applications.

### 4.1.1 Inertia weight

Inertia weight $\omega$ controls the momentum of the particle (Eberhart and Shi, 2001; Eberhart and Kennedy, 1995). Here two kinds of source geometry are adopted to evaluate more suitable ranges of parameters in the Tuned-PSO 2 nning of inertia weight, $0,0.4,0.7,0.9$, has been taken for two different acceleration coefficients at 1.4 and 2.0 respectively. From Figure 1, it is clear that the best convergence has performed by algorithm at inertia weight 0.7.This value of inertia weight produces high convergence rate at less number of iteration than the other values.

### 4.1.2 The maximum velocity $v_{\text {max }}$

The maximum velocity $\mathrm{v}_{\max }$ determines the maximum change one particle can undergo in its positional coordinates during iteration and used to avoid explosion and divergence. Usually, the full search ranges of the particle's positions as the vmax are fixed. For example, in case, a particle has position vector $\mathrm{x}=\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}\right)$ and if $-15<=\mathrm{x}_{\mathrm{i}}<=15$ for $\mathrm{i}=1,2$ and 3, then $\mathrm{v}_{\max }=$ 30 is fixed.

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How the inertia weight parameter has been decided. pl provide the reference.
As figure 1 , at iterations 50 to 100 all assigned ( $0,0.4,0.7,0.9$ ) value is convergence well? author should check the figure 1 .

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### 4.1.3 The swarm size

It is quite a common practice in the PSO literature to limit the range of number of particles.
Van den Bergh and Engelbrecht have shown that though there is a slight improvement of the optimal value with increasing swarm size, a larger swarm increases the number of function evaluations to converge to an error limit. However, Eberhart and Shi Illustrated that the population size has hardly any effect on the performance of the PSO method. So, in this paper population size has taken 100 .

### 4.1.4 The acceleration coefficients $c_{1}$ and $c_{2}$

To find the best tuning of learning parameters, various values of $\mathrm{c}_{1}, \mathrm{c}_{2}$ (i.e. $\mathrm{c}_{1}=\mathrm{c}_{2}=1.0,1.2$, $1.4,1.6,1.8$, and 2.0 ) and inertia weights (i.e. $0.4,0.7$ and 0.9 ) are taken, and various exercises have been made using the two different geometrical bodies by fixing the each of the inertia weight (Table 1). The results was analysed and found that more suitable values of $c_{1}$ and $c_{2}$ (i.e. $c_{1}=c_{2}=1.4$ ) are the best tuned acceleration coefficients for our case. These values of acceleration coefficients have been used to invert the gravity anomalies, which provide significant improvement and produce optimal solutions of the geological bodies.

### 4.2 Application to Synthetic gravity anomalies

Initially two geometrical models i.e. sphere and vertical cylinder has been considered for testing the applicability and efficacy of Tuned-PSO. The efficacy of proposed algorithm in terms of RMS error versus iterations is as shown in Figure 1. The gravity anomalies over these models are computed from equation (1) for the model parameters as shown in Table $1(\mathrm{a}, \mathrm{b})$ and $2(\mathrm{a}, \mathrm{b})$. In each case, the length of gravity profile of 51 km has 51 data points at one km equal interval. 2 年左avity anomaly for every source model is corrupted with $10 \%$ of white gaussian noise and Tuned-PSO based inversion algorithms applied on them. The optimized results obtained by 3 ned-PSO algorithms for synthetic noise free and with $10 \%$ noisy data. $\equiv \frac{4}{}$ igure 1 shows that tuned-PSO has best results at values $1.4,1.4$ and 0.7 for $\mathrm{c}_{1}, \mathrm{c}_{2}$ and inertia weight (w) respectively. This also shows that Tuned-PSO curve is having less number of local minima than other values. It means that the Tuned-PSO technique minimise the number of local minima for solving the geophysical nonlinear inverse problems.

The 5 mulated gravity anomaly by Tuned-PSO and computed gravity anomaly are shown in $\bar{\equiv}$ Figure 2(a) and 3(a) respectively and correspond to 7 nthetic gravity anomaly and 8 mputed 910maly corrupted with $10 \%$ of white gaussian noise as shown in Figure 2(b) and 3(b).

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Number: $1 \quad$ Author: sanjay Subject: Sticky Note Date: 9/28/2016 1:58:13 PM what is mean by 10\% corrupted and tunes-PSO based inversion applied.

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Number: $4 \quad$ Author: sanjay Subject: Sticky Note Date: 9/29/2016 12:02:49 PM what do u mean by synthetic noise free with $10 \%$ noisy data. rewrite the sentence.

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simulated and synthetic are different is meaning? computed gravity anomaly due to what?

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Figure 2 and Figure 3 show the well matching between the 2 rnetic curves and Tuned-PSO calculated gravity anomalies curves over spherical model and vertical cylindrical model respectively. Figure 4 shows the behaviour of $p_{\text {best }}$ and $g_{\text {best }}$ variation inside the algorithm and $3{ }^{\text {ts }}$ that $g_{\text {best }}$ decreases more rapidly toward the minimal error with high convergence. 4 e observed from Table 2 and 3 that the RMS error increases with increasing the noise in gravity anomaly however, the horizontal location $\left(\mathrm{x}_{0}\right)$ is a substantially stable parameter and varies in a small scale.

### 4.3 Application to Field gravity anomalies

### 4.3.1. Mobrun Sulphide Body, Near Rouyn- Noranda, Canada

Mobrun polymetallic deposit near Rouyn- Noranda comprises two complexes of massive lenses with mainly felsic volcanic rocks of the Archean Blake River Group (Barrett. et al., 1992). 6 fe main lens contents mainly massive sulphide, approximately 3.37 Million Ton with some other elements in least amount in comparison to sulphide are 0.95 Million Ton at $0.81 \% \mathrm{Cu}, 2.44 \% \mathrm{Zn}, 30.3 \mathrm{~g} / \mathrm{t} \mathrm{Ag}$, and $2.2 \mathrm{~g} / \mathrm{t} \mathrm{Au}$ estimated in 1989. 気 700 complex is located at 250 m to southeast of the Main complex. Host volcanic rocks of main complex are mostly massive, breciated, and tuffaceous rhyolites. Mobrun ore body is located at shallow depth; top of the body approximately 30 m depth and extended to 175 m . $\overline{\overline{=}}$

Tuned-PSO in MATLAB environment has been applied to field residual gravity anomaly. This anomaly profile of length 268 m has been taken from the Mobrun sulphide body, Noranda, Canada (Nettleton, 1976; Essa, 2012). It is seen from Figure 5 that both curves, $9{ }^{\text {nalysed }}$ from Tuned-PSO and observed gravity anomalies are extremely well correlated with optimal RMS error of $0.0271 \%$. The results in terms of model parameters (amplitude coefficient factor, shape factor and depth) over the Mobrun ore body analysed from Tuned-PSO method can seen in Table 4(a). This table provides the optimum results obtained from Tuned-PSO with $0.0271 \%$ error agrees well with the results obtained from
 Mobrun sulphide ore body reflects the shape of a semi-infinite vertical cylindrical geological body is present at depth of $30 \mathrm{~m}_{2} \boldsymbol{1 1}^{\text {an }}$ be seen from Table (b) $\mathbf{x}_{12}$ values of amplitude coefficient factor, shape factor and depth correspond to $60.0,0.77$ and 30 are more stable and consistent with results analysed from various authors, 13

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Number: 1 Author: sanjay Subject: Sticky Note Date: 9/29/2016 12:13:10 PM what is meaning of synthetic curves. is author has generated some curves? in place of figure 2 and 3 write $2 \mathrm{a}, \mathrm{b}$ and $3 \mathrm{a}, \mathrm{b}$.

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Number: 3 Author: sanjay Subject: Sticky Note Date: 9/29/2016 12:22:50 PM
Flase Statement. In table 2, no where mention about noise level as well as rms is constant upt to three decimal.

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Number: 5 Author: sanjay Subject: Sticky Note Date: 9/29/2016 12:24:44 PM
The sentence is not relevant to present studies. author should remove the sentence.

Number: 6 Author: sanjay Subject: Highlight Date: 9/29/2016 12:23:41 PM

Number: 7 Author: sanjay Subject: Sticky Note Date: 9/29/2016 12:25:37 PM
what is meaning of complex.
Rewirte the sentence.
Number: 8 Author: sanjay Subject: Sticky Note Date: 9/29/2016 12:26:37 PM
Reference is missing.
Number: 9 Author: sanjay Subject: Replacement Text Date: 9/29/2016 12:27:32 PM anomalies.

Number: 10 Author: sanjay Subject: Sticky Note Date: 9/29/2016 12:32:29 PM if there present result match well with other technique then what is the advantage of using Tuned PSO technique.
is it really worthy to resolve the depth of ore body. if so then up to what resolution. can this technique resolve the depth of body less then one meter.

T Number: 11 Author: sanjay Subject: Replacement Text Date: 9/29/2016 12:33:32 PM Rewrite the sentence.

F Number: 12 Author: sanjay Subject: Replacement Text Date: 9/29/2016 12:33:49 PM which table no.

国 Number: 13 Author: sanjay Subject: Replacement Text Date: 9/29/2016 12:34:49 PM if it is consistent then what is merit of present technique?

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### 4.3.2. Louga Anomaly West coast of Senegal, West Africa

The study area Louga anomaly of west coast of Senegal is taken for another case study for interpretation of gravity data using Tuned -PSO. The Senegal basin is part of the north-west African coastal basin- a typical passive margin basin opening west to the Atlantic. The complexities of the rift tectonics of the Atlantic opening gave rise to a series of sub-basins aligned north-south. The pre-rift (Upper Proterozoic to Palaeozoic), syn-rift (Permian to Lower Jurassic) and post-rift are divided into a number of sub-basins, controlled by east west transform related lineaments (Nettleton, 1962). In this paper Tuned-PSO in MATLAB environment has been also applied to another field case study. Gravity anomaly of Louga area, West coast of Senegal, West Africa (Essa, 2014) has taken for Tuned- PSO analysis as shown in Figure 6 has Profile length 32 km . The results in terms of model parameters (amplitude coefficient factor, shape factor and depth) over the Louga anomaly analysed from Tuned-PSO method can seen in Table 5(a). It is seen from Figure 6 that both curves $1^{\text {plysed }}$ from Tuned-PSO and observed gravity anomalies are extremely well correlated with optimal RMS error of $0.0271 \%$. The optimum obtained results of model parameters amplitude coefficient factor (A), shape factor ( q ) and depth ( z ) are $545.30 \mathrm{mGal}, 0.53$ and 4.92 km respectively that shows significantly good agreement with the results obtained by various authors as shown in Table 5(b). The Tuned PSO analysed value of shape factor confirms that the shape of the causative body is semi-infinite vertical cylindrical body present at depth about 4.92 km .

## 5. Conclusions

In this paper, various synthetic gravity anomalies and field gravity anomalies have been adopted for evaluating the applicability and efficacy of Tuned-PSO algorithms and also determining the suitable ranges of learning parameters setting (i.e. inertia weight and acceleration coefficients). On the basis of the performance, a novel algorithm PSO with suitable learning parameters has been implemented to gravity anomalies assuming models with gravity source geometry such as sphere and vertical cylinder. This technique has been tested and demonstrated on synthetic gravity anomalies with and without gaussian noise and finally applied to field residual gravity anomalies over Mobrun sulphide ore body, Noranda, QC, Canada and Louga Anomaly of West coast of Senegal, West Africa. This technique provides robust and plausible results even in the presence of noise that are consistent with the results obtained from other classical methods. Thus this technique is powerful tool that

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国 $\frac{\text { Number: } 1}{}$ anomalies $\quad$ Author: sanjay Subject: Replacement Text $\quad$ Date: 9/29/2016 12:36:08 PM anomalies

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improves the results of classical PSO and other technique significantly with less time and optimal error.

## 6. References

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## Figure and Table Captions

Figure 1. Iteration versus RMS error plot at different acceleration coefficients and inertia weights.
Figure 2. (a) Synthetic gravity anomaly versus Tuned-PSO calculated gravity anomaly over spherical model and (b) Synthetic gravity anomaly versus Tuned-PSO calculated gravity anomaly over same model with $10 \%$ white gaussian noise.

Figure 3. (a) Synthetic gravity anomaly versus Tuned-PSO calculated gravity anomaly over vertical cylindrical model, (b) Synthetic gravity anomaly versus Tuned-PSO calculated gravity anomaly over same model with $10 \%$ white gaussian noise.

Figure 4. Iteration versus RMS error of Tuned-PSO showing $p_{\text {best }}$ and $g_{\text {best }}$ over synthetic gravity anomaly.
Figure 5. Observed field gravity anomaly versus Tuned-PSO calculated gravity anomaly over Mobrun sulphide ore body, Canada.

Figure 6. Observed field gravity anomaly versus Tuned-PSO calculated gravity anomaly over West Senegal anomaly, Louga area, South Africa.

Table 1. Performance of the acceleration coefficients $c_{1}$ and $c_{2}$ using the synthetic gravity anomalies over spherical and vertical cylindrical geometrical bodies.
Table 2. (a) Optimized model parameters, converged iteration and RMS error in the inversion of synthetic gravity anomaly over a spherical source model and (b) optimized parameters, converged iteration and RMS error in the inversion of synthetic gravity anomaly with $10 \%$ white gaussian noise over a same source model from Tuned-PSO.

Table 3. (a) Optimized model parameters, converged iteration and RMS error in the inversion of synthetic gravity anomaly over a vertical cylindrical source model and (b) optimized parameters, converged iteration and RMS error in the inversion of synthetic gravity anomaly with $10 \%$ white gaussian noise over a same source model from Tuned-PSO.

Table 4. (a) Analysed results and parameters (A, z and q) used to invert the gravity anomaly over Mobrun sulphide ore body and (b) comparative results over Mobrun field, Canada from various methods and Tuned- PSO.

Table 5. (a) Analysed results and parameters (A, z and q) used to invert the gravity anomaly over West Senegal anomaly, Louga area, South Africa and (b) comparative results over same area from various methods and Tuned- PSO.

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Figure 1


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Figure 2(a)


Discussions
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Figure 2(b)


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Figure 3(a)


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Figure 3(b)


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Figure 4

Iteration vs RMS Error


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Figure 5


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Figure 6


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## (c) (i)

449 Table 1

| Gravity data <br> description | Weighting <br> factor | $\mathrm{c}_{1}=1.0$, <br> $\mathrm{c}_{2}=1.0$ | $\mathrm{c}_{1}=1.2$, <br> $\mathrm{c}_{2}=1.2$ | $\mathrm{c}_{1}=1.4$, <br> $\mathrm{c}_{2}=1.4$ | $\mathrm{c}_{1}=1.6$, <br> $\mathrm{c}_{2}=1.6$ | $\mathrm{c}_{1}=1.8$, <br> $\mathrm{c}_{2}=1.8$ | $\mathrm{c}_{1}=2.0$, <br> $\mathrm{c}_{2}=2.0$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | RMS Error |  |  |  |  |  |  |
| Synthetic <br> spherical <br> body | $\mathrm{w}=0.4$ | 0.004899 | 0.002899 | 0.00014 | 0.000907 | 0.000853 | 0.000861 |
|  | $\mathrm{w}=0.7$ | 0.002532 | 0.000118 | 0.000013 | 0.000087 | 0.000187 | 0.000247 |
|  | $\mathrm{w}=0.9$ | 0.005215 | 0.000118 | 0.000063 | 0.000379 | 0.000167 | 0.002695 |
| Synthetic <br> vertical <br> Cylindrical <br> body | $\mathrm{w}=0.4$ | 0.004892 | 0.003231 | 0.000327 | 0.000835 | 0.000704 | 0.000932 |
|  | $\mathrm{w}=0.7$ | 0.001913 | 0.000318 | 0.000011 | 0.000065 | 0.000207 | 0.000511 |

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Table 2

| (a) Optimized Parameters, converged iteration and RMS error in the inversion of synthetic <br> gravity anomaly over a spherical source model. |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Z (km) | A (mGal* $\left.\mathrm{km}^{2}\right)$ | q | $\mathrm{g}_{0}(\mathrm{mGal})$ | $\mathrm{x}_{0}(\mathrm{~km})$ | Iteration | RMS Error (\%) |
| 4.99883 | 550 | 1.5 | 24.0 | $-1.89 \times 10^{-3}$ | 100 | 0.000405 |
| 4.9999 | 660.31 | 1.5 | 24.0 | $2.39 \times 10^{-5}$ | 200 | 0.000015 |
| 5.00 | 610.15 | 1.5 | 24.0 | $-1.44 \times 10^{-6}$ | 300 | 0.00 |
| 5.00 | 604.36 | 1.5 | 24.0 | $3.3 \times 10^{-13}$ | 400 | 0.00 |
| 5.00 | 604.10 | 1.5 | 24.0 | $8.17 \times 10^{-16}$ | 500 | 0.00 |

(b) Optimized Parameters, converged iteration and RMS error in the inversion of synthetic gravity anomaly with $10 \%$ white guassian noise over a spherical source model.

| $\mathrm{z}(\mathrm{km})$ | $\mathrm{A}\left(\mathrm{mGa}{ }^{*} \mathrm{~km}^{2}\right)$ | q | $\mathrm{g}_{0}(\mathrm{mGal})$ | $\mathrm{x}_{0}(\mathrm{~km})$ | Iteration | RMS Error (\%) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4.5 | 605.49 | 1.5 | 24 | $-6.95 \mathrm{X10}^{-2}$ | 100 | 0.174890 |
| 4.5 | 603.99 | 1.5 | 24 | $-6.81 \mathrm{X10}^{-2}$ | 200 | 0.174885 |
| 4.5 | 550.32 | 1.5 | 24 | $-6.86 \mathrm{X10}^{-2}$ | 300 | 0.174883 |
| 4.5 | 601.42 | 1.5 | 24 | $-6.86 \mathrm{X10}^{-2}$ | 400 | 0.174883 |
| 4.5 | 680.0 | 1.5 | 24 | $-6.85 \mathrm{X10}^{-2}$ | 500 | 0.174883 |

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Table 3
(a) Optimized Parameters, converged iteration and RMS error in the inversion of synthetic gravity anomaly over a vertical cylindrical source model.

| $\mathrm{Z}(\mathrm{km})$ | $\mathrm{A}\left(\mathrm{mGal} \mathrm{km}^{2}\right)$ | q | $\mathrm{g}_{0}(\mathrm{mGal})$ | $\mathrm{x}_{0}(\mathrm{~km})$ | Iteration | RMS Error (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3.015 | 182.31 | 0.5 | 66.33 | $-4.4 \times 10^{-3}$ | 100 | 0.001743 |
| 3.016 | 185.92 | 0.5 | 66.33 | $-3.7 \times 10^{-4}$ | 200 | 0.001635 |
| 3.016 | 192.59 | 0.5 | 66.33 | $-2.74 \times 10^{-10}$ | 300 | 0.001633 |
| 3.015 | 162.15 | 0.5 | 66.33 | $-7.98 \times 10^{-11}$ | 400 | 0.001633 |
| 3.016 | 169.00 | 0.5 | 66.33 | $-6.58 \times 10^{-11}$ | 500 | 0.001633 |

(b) Optimized Parameters, converged iteration and RMS error in the inversion of synthetic gravity anomaly with $10 \%$ white guassian noise over a vertical cylindrical source model.

| $\mathrm{Z}(\mathrm{km})$ | $\mathrm{A}\left(\mathrm{mGa}{ }^{*} \mathrm{~km}^{2}\right)$ | q | $\mathrm{g}_{0}(\mathrm{mGal})$ | $\mathrm{x}_{0}(\mathrm{~km})$ | Iteration | RMS Error (\%) |
| :---: | :---: | :---: | :---: | :--- | :--- | :---: |
| 3.02 | 167.33 | 0.5 | 65.81 | $-3.95 \times 10^{-2}$ | 100 | 0.036732 |
| 2.99 | 160.38 | 0.5 | 66.33 | $1.36 \times 10^{-2}$ | 200 | 0.036968 |
| 3.02 | 161.74 | 0.5 | 65.88 | $-4.52 \times 10^{-2}$ | 300 | 0.036672 |
| 30.2 | 160.35 | 0.5 | 65.88 | $-4.50 \times 10^{-2}$ | 400 | 0.036672 |
| 3.02 | 198.67 | 0.5 | 65.88 | $-4.50 \times 10^{-2}$ | 500 | 0.036672 |

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Table 4
(a) Optimized Parameters, converged iteration and RMS error in the inversion of field gravity anomaly over Mobrun sulphide ore body.

| $\mathrm{z}(\mathrm{km})$ | $\mathrm{A}\left(\mathrm{mGal} \mathrm{km}^{2}\right)$ | q | $\mathrm{g}_{0}(\mathrm{mGal})$ | $\mathrm{x}_{0}(\mathrm{~km})$ | Iteration | RMS Error (\%) |
| :---: | :---: | :---: | :---: | :---: | :--- | :---: |
| 31 | 58.08 | 0.77 | 1.7781 | -2.99078 | 100 | 0.027149 |
| 31 | 59.55 | 0.76 | 1.1156 | -3.02429 | 200 | 0.027163 |
| 31 | 58.00 | 0.76 | 1.7826 | -2.13091 | 300 | 0.027125 |
| 31 | 59.03 | 0.77 | 1.7826 | -2.15033 | 400 | 0.027124 |
| 30 | 59.99 | 0.77 | 1.7992 | -2.15013 | 500 | 0.027124 |

(b) Comparative results over Mobrun field example from various methods and GPSO.

| Parameter | Grant and West <br> $(1965)$ | Euler deconvoltuion <br> (Roy et al., 2000) | Fast interpretation <br> Method | Tuned- PSO <br> Method |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Z}(\mathrm{m})$ | 30 | 29.44 | 33.3 | 30.0 |
| q | - | 0.77 | 0.78 | 0.77 |
| $\mathrm{~A}(\mathrm{mGal})$ | - | - | 59.1 | 60.0 |

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Table 5

| (a) Optimized Parameters, converged iteration and RMS error in the inversion of field <br> gravity anomaly over West Senegal (Louga area) anomaly. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{z}(\mathrm{km})$ | $\mathrm{A}\left(\mathrm{mGal}^{*} \mathrm{~km}^{2}\right)$ | q | $\mathrm{g}_{0}(\mathrm{mGal})$ | $\mathrm{x}_{0}(\mathrm{~km})$ | Iteration | RMS Error (\%) |
| 4.90 | 549.44 | 0.52 | 94.83 | $-2.60 \times 10^{-1}$ | 100 | 0.027065 |
| 4.90 | 550.0 | 0.53 | 94.80 | $-2.56 \times 10^{-1}$ | 200 | 0.026552 |
| 4.91 | 549.57 | 0.53 | 94.79 | $-2.45 \times 10^{-1}$ | 300 | 0.026552 |
| 4.91 | 547.66 | 0.53 | 94.79 | $-2.42 \times 10^{-1}$ | 400 | 0.026551 |
| 4.91 | 545.30 | 0.53 | 94.79 | $-2.39 \times 10^{-1}$ | 500 | 0.025551 |

(b) Comparative results of various methods over West Senegal (Louga area) anomaly.

| Parameter | New fast least square method (Essa, 2014) | Tuned-PSO method |
| :---: | :---: | :---: |
| $\mathrm{z} \mathrm{(km)}$ | 4.94 | 4.92 |
| q | 0.53 | 0.53 |
| $\mathrm{~A}(\mathrm{mGal})$ | 545.68 | 545.30 |

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