



1     **Appraisal on inversion algorithm techniques in 2D electrical resistivity tomography**  
2                     **survey data for poised mapping of subsurface features**

3             Abhay Kumar Bharti\*, Amar Prakash and Krishna Kant Kumar Singh  
4             CSIR-Central Institute of Mining and Fuel Research, Dhanbad, India- 826001

5                     \*Corresponding author's email: [abhay.bharati@gmail.com](mailto:abhay.bharati@gmail.com)

6     **Abstract:**

7     Analysis of non-uniqueness model in resistivity imaging data is vital in inaugurating the  
8     consistency of models. Nevertheless, such analysis is moderately unusual in resistivity  
9     imaging data set. Electrical resistivity tomography (ERT) technique is being constantly used  
10    in many scientific areas including engineering, environmental and archaeological survey.  
11    Primarily, the inversion algorithm techniques are employed on synthetic model data set with  
12    and without some random Gaussian noise, and its validity is tested by filed data set. The  
13    study was conducted in the premises of Central Institute of Mining and Fuel Research  
14    (CIMFR), Dhanbad by laying an ERT profile of 480 m length with 5 m electrode spacing  
15    using Syscal Pro (Iris instrument) resistivity meter. Two standard arrays were used in this  
16    study namely Wenner-Schlumberger and dipole-dipole array. The data set was mixed to a  
17    single array to achieve better resolution and enhanced clarification. On processing data by  
18    Prosys-II software, it was exported in Res2Dinv software for inversion. In this context, data  
19    was inverted by different algorithm techniques i.e. least square ( $L_2$ -norm) and robust  
20    inversion ( $L_1$ -norm). Exemplary results related to the heterogeneity of the resistivity structure  
21    within the high and low resistivity anomaly were obtained by robust inversion method. The  
22    obtained results are in broad agreement with the simulation model.

23    **Keywords:** *ERT, Least square, Robust inversion, CSIR-CIMFR, Existing structures*

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26 **1.0 Introduction:**

27 A significance of ground based geophysical technique is related to as much information as  
28 imaginable on subsurface existing structures and composition of materials which remain in  
29 the subsoil. All surface geophysical techniques are function of physical properties of the earth  
30 materials. Changes in subsurface properties such as porosity, permeability, density, saturation  
31 of water etc. may be distinguished by geophysical survey like gravity, seismic, and electrical  
32 methods (e.g., Ezersky 2008; Keller and Frischknecht 1996). Currently, it is common to  
33 apply geophysical techniques to environmental, engineering and mining related problems at  
34 shallow depths and it is valid solution for target identification at complex subsurface  
35 structures.

36 A non-invasive surface geophysical technique such as Electrical Resistivity Tomography  
37 (ERT) belongs to the family of the most applied geophysical methods in an extensive  
38 spectrum of mapping of near surface problems and environmental studies (e.g., Singh et al.,  
39 2004; Dahlin and Zhou 2006; Chandra et al., 2008; Kumar 2012; Singh 2013b; Bharti et al.,  
40 2016a,b; Bharti et al., 2019). This technique has become the most routinely used geoelectrical  
41 application for delineating the complex geological features of subsurface of the earth due to  
42 its comparative effortlessness and time effectivity. A better understanding of the subsurface  
43 geoelectrical structures in hard rocky terrain can be achieved by this technique. Electrical  
44 properties of the earth mass at shallow depth can be obtained by 2D ERT technique in both  
45 vertical and horizontal orientations, which helps in notching up of status of strata in  
46 qualitative and quantitative forms.

47 The geoelectric distinction between dissimilar natures of earth constituents is a reasonable  
48 means to classify different material characteristics, which has been allocated to the degree of  
49 weathering, moisture content and mineralogical composition of such earth material. A cell-  
50 based inversion technique is usually implemented for effectively prototypical complicated



51 structures along an uninformed resistivity spreading in subsurface of the earth. Therefore, this  
52 procedure makes numerous rectangular cells with fixed positions and sizes by division of  
53 subsurface block.

54 A few researchers have done work over comparison of inversion techniques in 2D resistivity  
55 data sets. For example, the work done by Loke (2003), encompasses analysis on smooth and  
56 blocky inversion methods in 2D resistivity survey. According to them, better results are  
57 obtained by smooth inversion method in which change in resistivity is gradual; on the other  
58 hand outcomes of blocky inversion method gives significant results for sharp boundaries. The  
59 study was carried out over karstic structures by Hamdan and Vafidis (2009), by inversion  
60 techniques for eminent image of resistivity. Three different inversion methods i.e. combined,  
61 smoothness constrained and robust inversion were adopted on real data set and results were  
62 compared and also combined inversion of two standard configurations namely, Wenner-  
63 Schlumberger and dipole-dipole was conducted to obtain the highest reliability of the 2D  
64 resistivity section.

65 The CSIR-CIMFR campus situated in Dhanbad area, India. Dhanbad, the coal capital of  
66 country, lies in the mid-eastern part of Jharkhand state. Dhanbad district is evaluated as dry  
67 because of deficiency of immense rivers and high temperature. The district is related to small  
68 scale of ponds and two big dams which are good medium to recharge groundwater.  
69 Therefore, groundwater arises in this zone below unconfined state in the weathered  
70 formations at low depths in utmost of the lithological components in the Achaean and nearly  
71 all the lithological components in the Gondwana formation. Groundwater arises below  
72 confined to semi confined state where the fractures are deep seated and are unrelated with  
73 the top weathered formation (e.g., Kumar 2018).

74

75



76 **2.0 Methodology:**

77 In general, inversion procedure is involved to renovate the real circulation of acquired  
78 resistivity data sets as the latter does not deliver anticipated facts. The present study was  
79 conducted over the premises of Central Institute of Mining and Fuel Research, Dhanbad,  
80 India as shown in Fig. 1. Keeping in view the literature review in background, the scope of  
81 study encases analysis of two different inversion methods, namely, least square ( $L_2$ -norm)  
82 and robust inversion ( $L_1$ -norm) in 2D resistivity data set for mapping of complex subsurface  
83 existing structures. The idea of multiple inversion techniques could be used for evaluating the  
84 superiority of true 2D resistivity models. Inversion technique is a procedure to create a model  
85 that clarifies a set of measurements. It is related to make direct assumptions about the earth  
86 from DC resistivity measurements due to the contests of envisaging large data sets (e.g., Loke  
87 et al., 2003).

88 **2.1 Synthetic Model:**

89 Initially, the comparison of two different algorithm techniques i.e. least square and robust  
90 inversion in 2D electrical resistivity data set to map the complex subsurface existing  
91 structures through forward modelling, considered for better interpretation of field data set  
92 using RES2DMOD software package (e.g., Loke and Barker, 1996).

93 In this context, the model was consisted of four homogeneous layers i.e. (i) soil/alluvium  
94 layer, (ii) semi weathered rock layer, (iii) hard weathered rock layer and (iv) bed rock/  
95 basement rock layer where their apparent resistivity values are of 100  $\Omega\text{m}$ , 300  $\Omega\text{m}$ , 500  $\Omega\text{m}$ ,  
96 and 1000  $\Omega\text{m}$  with 64 equally spaced electrodes with 5m interval using finite difference  
97 algorithm technique. Finite difference algorithm technique divides the model subsurface into  
98 a number of rectangular blocks (e.g., Loke et al., 2003). Two conductive body (resistivity  
99 ranging 10  $\Omega\text{m}$  to 100  $\Omega\text{m}$ ) and one resistive body (resistivity ranging 1000  $\Omega\text{m}$  to 2000  $\Omega\text{m}$ )  
100 was incorporated in model set.



101 The simulated resistivity retorts of the section were initiated using Wenner-Schlumberger,  
 102 dipole-dipole and combined inversion of both arrays with and without some random Gaussian  
 103 noise added to validate field condition and get more representative results. The synthetic  
 104 apparent resistivity model data set was inverted by using RES2DINV software for producing  
 105 true resistivity variation of subsurface of the earth.

106 Figure 2 and 3 shows the obtained outcomes from Wenner-Schlumberger, dipole-dipole and  
 107 combined inversion of both arrays using least square and robust inversion algorithm  
 108 techniques. Stimulatingly, all outcomes recover the anomaly locations through both inversion  
 109 techniques. However, in robust inversion technique was recognized both depth and  
 110 extensions of anomaly in all inverted resistivity models with greater resolution compared to  
 111 least square technique. It is also observed that the combined inversion of both arrays gives  
 112 the better results with high resolution compared to Wenner-Schlumberger and dipole-dipole  
 113 array. For example, Dahlin and Zhou (2004), reported that the imaging with combined arrays  
 114 generates models similar to the preferable observation model among the specific array.

## 115 **2.2 Smooth-constrained least-squares technique:**

116 This technique usually uses the form of regularised least-squares optimization method in the  
 117 smooth-constrained or  $L_2$ -norm. The mathematical expression of this technique (e.g.,  
 118 deGroot-Hedlin and Constable 1990; Ellis and oldenburg 1994) is expressed as:

$$119 \quad (J_i^T J_i + \lambda_i W^T W) \Delta r_i = J_i^T g_i - \lambda_i W^T W r_{i-1} \quad (1)$$

120 Where,  $g_i$  = data misfit vector,

121  $\Delta r_i$  = change in the model parameters for the  $i$ th iteration,

122  $W$  = roughness filter,

123  $\lambda$  = damping factor,

124  $r_{i-1}$  = model parameters vector for the previous iteration and

125  $J$  = Jacobian matrix of partial derivatives.



126 Roughness is filtered by first-order finite difference operator (e.g., deGroot-Hedlin and  
 127 Constable 1990). The equation (1) helps in minimising the sum-of-squares of the data misfit  
 128 and sum-of-squares of the model roughness.

129 Smooth-constrained least-squares technique reduces the sum of squares of the spatial changes  
 130 in the model resistivity and the data misfit. Optimal results are obtained for geologically  
 131 smooth variation subsurface (e.g., Barker 1992). However, it shows spread boundaries for  
 132 sharp transition like igneous dyke.

### 133 **2.3 Robust or blocky inversion technique:**

134 The cumulated absolute value of spatial changes in resistivity model can be reduced by  
 135 Robust inversion technique. It is also known as  $L_1$ -norm measure of the data misfit (e.g.,  
 136 Claerbout and muir 1973). The mathematical formulations used by  $L_1$ -norm optimisation  
 137 method is

$$138 \quad (J_i^T R_d J_i + \lambda_i W^T R_m W) \Delta r_i = J_i^T R_d g_i - \lambda_i W^T R_m W r_{i-1} \quad (2)$$

139 Where,  $R_d$  and  $R_m$  = weighting matrices

140 Constant resistivity values of each part are produced on application of  $L_1$ -norm to model  
 141 roughness filter (e.g., Farquharson and Oldenburg 1998). Sharp boundary separation is also  
 142 obtained by this technique.

### 143 **3.0 Discussions:**

144 2D ERT section of profile AA' was generated by the configurations of Wenner-  
 145 Schlumberger, dipole-dipole and combined inversion of both arrays for the length of 480 m  
 146 with electrode interval of 5m using Syscal Pro (Iris instrument) resistivity meter with 96  
 147 electrodes (Fig.2). Least square and robust inversion technique was adopted for analysis of  
 148 subsurface existing geological formation using Res2Dinv handling software as shown in Figs.  
 149 3 & 4.

150



### 151 **3.1 Inverted geoelectrical section of Least square inversion of profile AA'**

152 The 2D geoelectric model of profiles AA' along with the least square inversion technique  
 153 projected using Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays  
 154 are shown in Figures 3a, b & c respectively. The outcomes obtained by electrical resistivity  
 155 tomography designate an extensive range of resistivity variation through the profile.  
 156 Topmost layer up to a depth of 10m consisting of soil/ alluvium having a resistivity of about  
 157 2 to 80  $\Omega\text{m}$  was considered for all ERT sections. Two water aquifers ( $L1Z2^{\text{ws}}$  &  $L2Z2^{\text{ws}}$ )  
 158 associated with fracture zones with relatively low resistivity of 2 to 12  $\Omega\text{m}$  at the surface  
 159 distance of about 130 m to 180 m and 280 m to 305m were delineated in 2D geoelectric  
 160 section generated by Wenner-Schlumberger array ( Fig.3a) and one water body ( $L2Z2^{\text{c}}$ ) was  
 161 also identified in combined inversion of both arrays along the surface distance at about 280 m  
 162 to 305m ( Fig.3c). Relatively high resistivity (230 to 608  $\Omega\text{m}$ ) anomaly associated with  
 163 weather rock / fracture rock ( $WZ2^{\text{dd}}$  &  $WZ2^{\text{c}}$ ) was identified along 2D ERT section of dipole-  
 164 dipole and combined inversion of both arrays at reduced distance (RD) of 25 to 90 m (Fig.3b  
 165 & c). A high resistivity contrast of more than 1600  $\Omega\text{m}$  associated with bed rock/ hard rock  
 166 ( $HZ3^{\text{ws}}$ ,  $HZ3^{\text{dd}}$  &  $HZ3^{\text{c}}$ ) was detected in all 2D resistivity sections (Wenner-Schlumberger,  
 167 dipole-dipole and combined inversion of both arrays) along a surface distance of 215 to 280  
 168 m.

### 169 **3.2 Inverted geoelectrical section of Robust inversion of profile AA'**

170 2D ERT inverse model of profiles AA' along with the robust inversion technique projected  
 171 using Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays are shown  
 172 in Figures 4a, b & c respectively. Wide range in resistivity was observed by this technique  
 173 also.  
 174 Top layer consisting of soil/ alluvium was encountered up to depth of 10 m followed by  
 175 Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays are shown in



Figures 4a, b & c respectively. A prominent signature of relatively low resistive ( $L2Z2^{ws}$ ,  $L2Z2^{dd}$ ,  $L2Z2^c$ ) water aquifer zone associated with fracture rock mass at RD of about 280 m to 305 m was identified with resistivity range of about 2 to 80  $\Omega m$  in all 2D ERT section models. In addition, one water body ( $L2Z2^{ws}$ ) was also identified in Wenner-Schlumberger array along the surface distance of about 280 m to 305m ( Fig.4a). A signature of weather rock / fracture rock ( $WZ1^{ws}$ ,  $WZ1^{dd}$  &  $WZ1^c$ ) was recognized along 2D ERT sections of Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays at RD of 25 to 110 m with moderately high resistivity range of 230 to 608  $\Omega m$  (Fig.4). The bed rock/ hard rock ( $HZ3^{ws}$ ,  $HZ3^{dd}$  &  $HZ3^c$ ) with high resistivity signature of more than 1600  $\Omega m$  was demarcated in all the 2D geoelectrical models of profile AA' projected by Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays at the surface distance of about 195 to 270 m.

The soil/ alluvium layer showed low resistivity up to 10 m depth by both the techniques. A signature of weather rock / fracture rock was delineated in least square inversion technique only for dipole-dipole and combined inversion of both arrays. However, in robust inversion technique this feature was visibly identified in all resistivity sections. The extension of water aquifer zone at greater depth associated with fracture rock mass was well demarcated by combined inversion of both arrays through  $L_1$ - norm in comparison to  $L_2$ -norm.

The outcomes generated of both synthetic and field conditions by inversion algorithm revealed that a combination of Wenner-Schlumberger and dipole- dipole array would provide maximum subsurface information and the optimal arrays sensitivity as this combination can encompass both strong signal/noise ratio and sensitivity to vertical and lateral changes. A prominent subsurface existing structure in geoelectrical sections by resistivity data sets could be assessed by comparing the outcomes of inversion techniques. This is vital particularly





200 where sudden resistivity changes like geologic interfaces characterized by variation in  
201 lithology are anticipated.

#### 202 **4.0 Conclusions:**

203 Initially, the synthetic data was generated using Res2Dmod software. Field situation was  
204 simulated through forward modelling. Two different algorithm techniques i.e. Least square  
205 inversion and Robust inversion were studied in 2D electrical resistivity data set for mapping  
206 of complex subsurface existing structures over a part of the CSIR-CIMFR campus using  
207 Wenner-Schlumberger, dipole- dipole and combined inversion of both arrays. Robust  
208 inversion indicates an additional feature with combined inversion of both arrays compared to  
209  $L_2$ -norm and it has good convergence throughout the iteration process, enabling easy  
210 analysis. The extension of aquifer zone associated with fracture rock mass at greater depth  
211 with high resolution was well demarcated by robust inversion indicates an additional feature  
212 with combined inversion of both arrays. A complex subsurface existing structure in  
213 geoelectrical sections by ERT data sets could be evaluated by comparing the consequences  
214 from the two inversion schemes.

215 **5.0 Data availability:** Outcomes are in the form of images shown in Figs.1, 2, 3, 4, 5 and 6.  
216 There is no data in addition.

217 **6.0 Team list:** Abhay Kumar Bharti, Amar Prakash and Krishna Kant Kumar Singh

#### 218 **7.0 Author contribution:**

219 Abhay Kumar Bharti: Conducted field investigation, data interpretation and preparation of  
220 manuscript.

221 Amar Prakash: Contributed in enhancing data interpretation and elevation in manuscript  
222 quality.

223 Krishna Kant Kumar Singh: Contributed in site selection for investigation and data  
224 interpretation.



225 **8.0 Competing interests:** Authors have not competing interest in any aspect.

226 **9.0 Disclaimer:** No such issue

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294 array, (b) dipole–dipole array and (c) combined inversion of both arrays with Robust  
295 inversion technique

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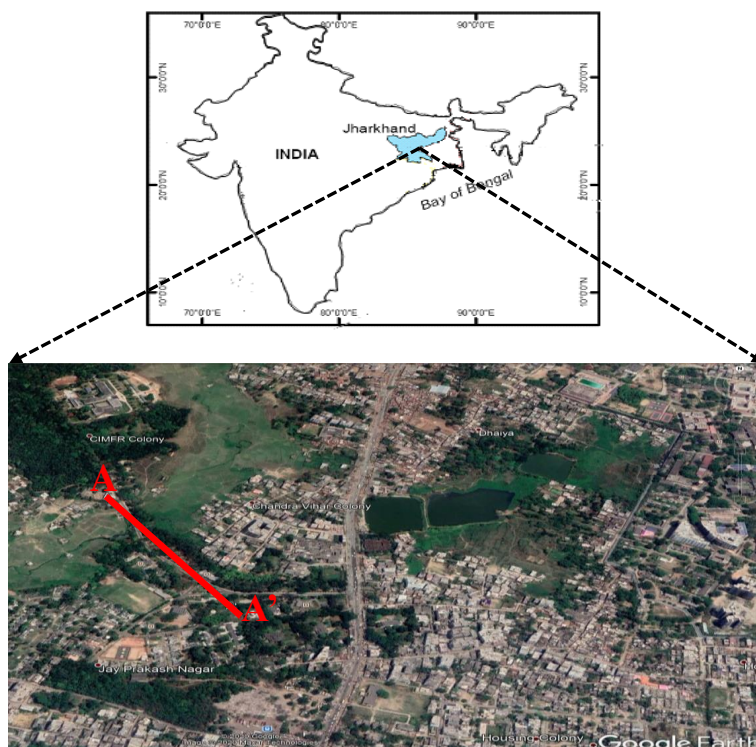
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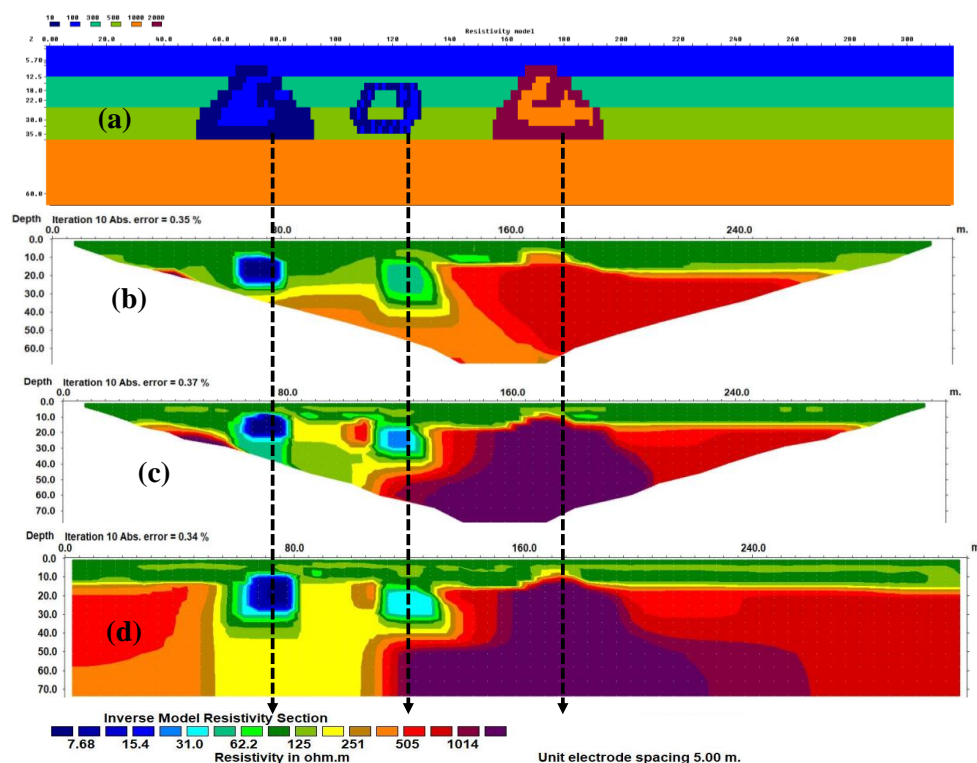
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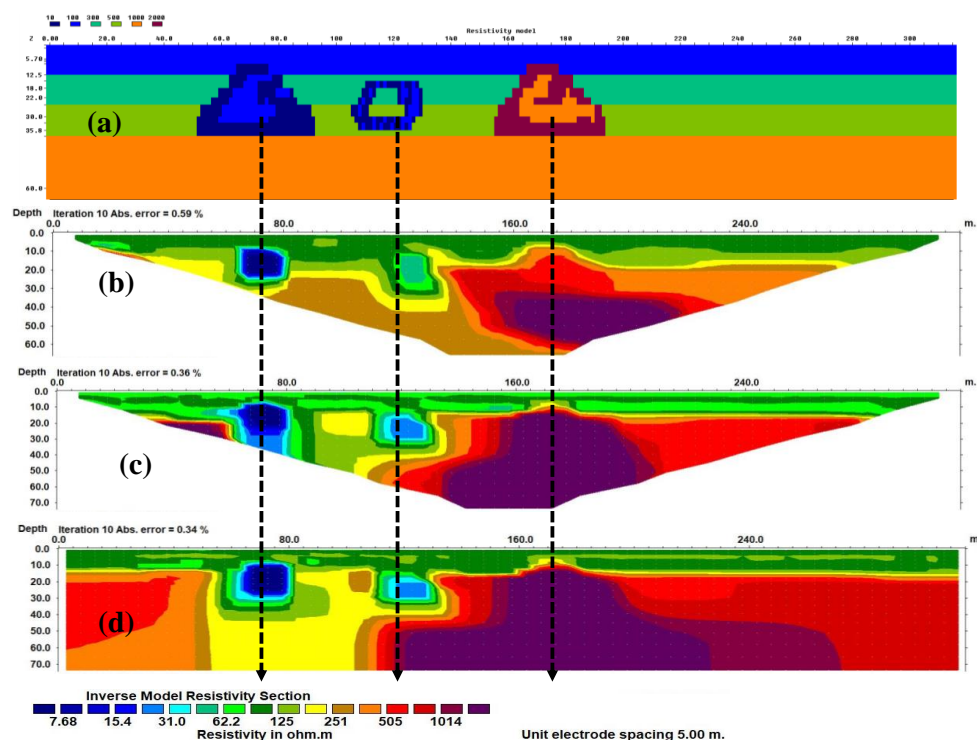
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**Figure.1:** Location map of the study area © Google Earth



**Figure.2:** Synthetic model outcomes (a) synthetic geological formation (b) inverted resistivity model of Wenner–Schlumberger array (c) inverted resistivity model of dipole–dipole array and (d) combined inversion of both arrays with Least square inversion technique



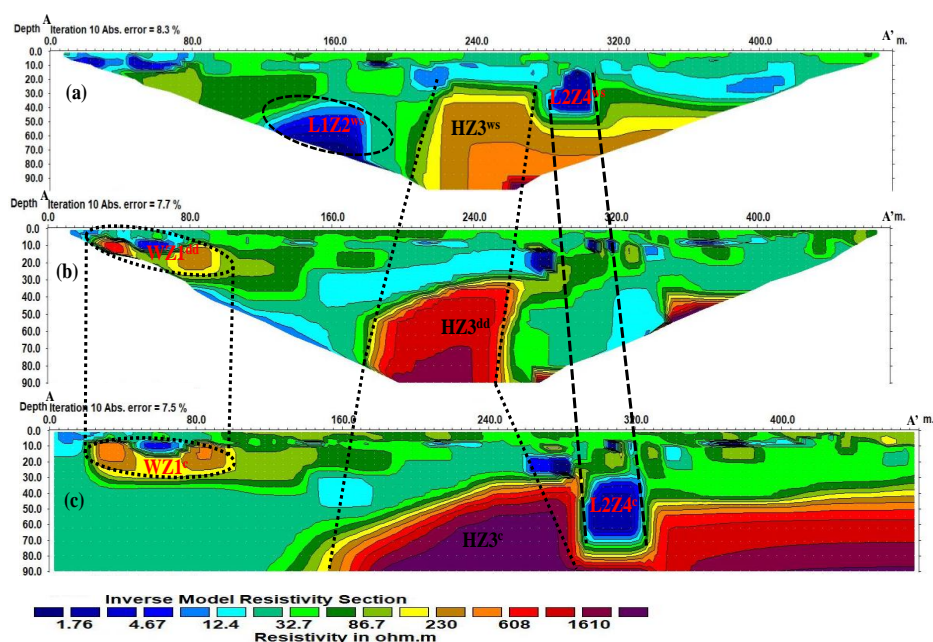
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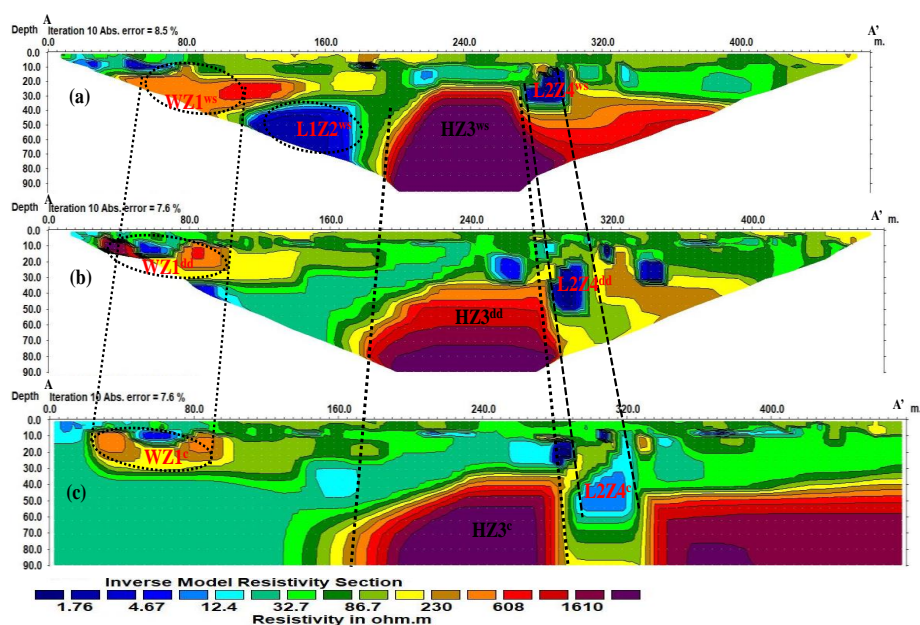


**Figure.4:** SYSCAL Pro-96 (Iris Instrument) data-acquisition field setup using 96 electrodes





**Figure.5:** 2D ERT section along profile AA' over the study area: (a) Wenner–Schlumberger array, (b) dipole–dipole array and (c) combined inversion of both arrays with Least square inversion technique



**Figure.6:** 2D ERT section along profile AA' over the study area: (a) Wenner-Schlumberger array, (b) dipole-dipole array and (c) combined inversion of both arrays with Robust inversion technique