

In this document, we provide the response for Reviewers 1 and 2. Some of the concerns raised by Reviewer 2 led us to eliminate parts of the paper. Hence, any comments from Reviewer 1 in those parts are not addressed here as they are not part of the paper anymore. The line numbers referred to below are the line numbers from the pre-print of the manuscript that was originally sent to the reviewers.

Response to Review 1

First, we would like to thank the reviewer for the comments and suggestions, which we think improved the quality of the paper. Below, we address all comments and concerns that were raised. The reviewer's comment is reiterated after the line number on which it occurred, and followed by our answer.

Line 35: "To make a measurement of the absolute electron density with any ISR an independent measurement is required." - Rephrase?

We have rephrased those sentences slightly so that it is clearer.

Line 35: "...plasmaline..." - two words?

Fixed for all instances of the word.

Line 90: 3 TYPOs

Fixed.

Line 105: "...a magic constant Δ ..." - I have not seen the Deathly Hallows symbol used for this stuff before. Although I think it is important to have fun as a scientist, as a reviewer, I want to point out that this symbol is not easy to reproduce, which may cause some frustrations with colleagues and in future works.

We have now changed the symbol for the magic constant to Υ , a standard Greek letter.

Line 180: Figure 3 - The title of the figure sounds like you are talking about one beam over the span of multiple days. Is this title misleading?

The title is correct. It is an example of one beam at a specific altitude during an experiment that lasted multiple days. NOTE: This is now Figure 1 in the final version.

Line 185: beam - And at a given time and altitude?

Added.

Line 190: "...all beams at the altitude of interest..." - How wide is the altitude of interest?

This was also addressed by Reviewer 2, and we have made changes and clarifications in the manuscript now.

Line 210: "...data quality parameters were poor..." - Elaborate?

We have clarified this in the manuscript.

Line 280: 2 TYPOs

Fixed.

Response to Review 2

We would like to thank the reviewer as well for their detailed and constructive feedback. Their suggestions resulted in multiple changes which were instrumental in improving the quality of the paper. We appreciate the time and effort taken to provide such thorough suggestions, which have significantly strengthened the work.

Remark 1

Thank you for pointing this out. At a radar frequency of 440 MHz, the Debye length effects become important for electron densities below $5 \cdot 10^{10} \text{ m}^{-3}$ (Vadas and Nicolls 2009). In this case, the ion-line power is no longer linearly dependent on electron density. Also, the shape of the ion-line also changes in a non-linear way. Figure 1 shows the effect on the ion-line shape, showing how at low densities the ion-line starts to vanish, and is replaced with a broader spectral feature that at sufficiently low densities is the Doppler spectrum of free electrons not affected by collective interactions.

The primary reason for not including the Debye length correction was the assumption that the majority for the data would be in the linear regime, where $4\pi D/\lambda \approx 0$. This also conveniently fits with adapting the flat-field methodology to radar imaging.

While the majority of the measurements e.g., in Figure 4 of the paper have $N_e > 5 \cdot 10^{10}$, lower densities are also included in the data. As you point out, the altitude variations could be due to this effect. For example, on the top-side and bottom-side of the ionosphere, the electron density will approach low enough densities that the Debye length effects become important.

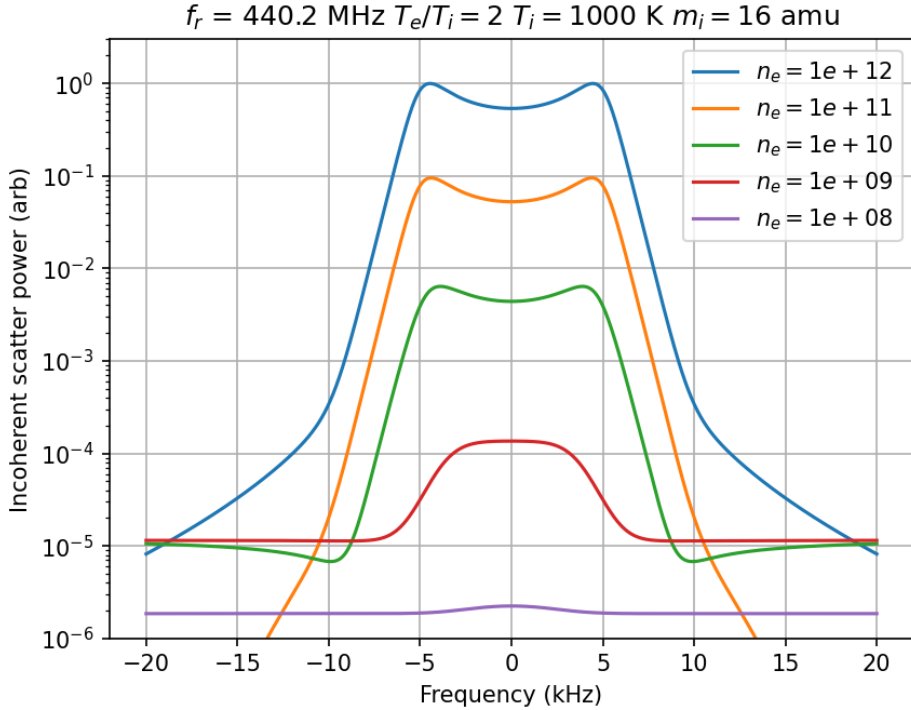


Figure 1: Ion-line shape as a function of electron density.

The bit/pulse length was left out, because we assumed it to be part of the magic constant. As we were making relative comparisons with different beams running the same experiment, τ_p is the same for all beams and is not important for describing relative power calibration variations from one beam to another.

In order to match the established AMISR calibration, we have modified the equation, also including the Debye length term. We have also added a discussion on Debye length effects.

Remark 2

The points you make are correct. The key point of the article is to: show that some AMISR experiments have errors in calibration that affects comparison of data across beams, and to show a simple method that can be used to perform a first order correction to electron density. This method is not perfect, but it already greatly improves uniformity of relative electron density measurements from one beam to another.

n_e (m ⁻³)	error %
10 ⁹	782.90
5 · 10 ⁹	100.50
10 ¹⁰	46.74
5 · 10 ¹⁰	8.79
10 ¹¹	4.36
5 · 10 ¹¹	0.87
10 ¹²	0.43

Table 1: Relative errors in ion-line power when using $1/(1 + T_e/T_i)$ instead of $1/[(1 + k^2\lambda_D^2)(1 + k^2\lambda_D^2 + T_e/T_i)]$. Assuming $T_e/T_i = 2$.

Taking into account the non-linearity that you point out would be far more involved and essentially require use of observed autocorrelation functions and performing the fits again from scratch. It is not obvious how including intercalibration across all beams for extended periods of time could be included into such a fit. Therefore, this is beyond what can be done in this article.

We have taken into account your comment by adding a discussion of this non-linearity, also pointing out that due to this non-linearity, low densities have correlated errors in electron density and the Te/Ti ratio.

We have calculated the error in the radar cross-section calculation for different electron densities and temperature ratios to point out the error in the simple approximation. The errors are shown in Table 1. We expect the error in ion-line power to be less than 9% for the majority of the data.

Remark 3

The purpose of the power measurement bias term η was to adapt the flat-field equation to incoherent scatter radar. The equation is a simplification, assuming that the majority of the FIR filter response is in the zero lag.

As you describe, incoherent scatter radar signal processing techniques take into account the shape of the FIR filter when subtracting receiver noise and when fitting the ion-line, doing this for all of the lags of the measured autocorrelation function.

It is correct that $\eta = 0$. In the first draft of the article, this is the value that flat-fielding methodology also found.

Based on your comment, we have removed the discussion on the term η .

Remark 4

In the calculation, we have used the nearest-neighbor altitude gate. This is done both for calculations using the fitted N_e and when using N_e inferred from the backscattered power (Ne_NoTr). Typical altitude offsets are 3 km for Ne_NoTr and 20 km for Ne from the fitted ACF. We have made this clearer in the manuscript now.

Remark 5

We agree that in a polar plot or some other two-dimensional projection, the terminator line should be smooth. In this plot the different beams are organized by their magnetic latitude, not magnetic latitude *and* longitude. Some beams that have similar magnetic latitude but have different longitude will be adjacent in this plot. Hence, the terminator is not expected to be smooth in this plot. Here it is mainly an indicator of the time of sunset and sunrise in each beam. We have clarified this in the manuscript now.

Remark 6

This is a good point. We have now also calculated the correction factors from the electron density inferred from backscattered power (Ne_NoTr). The results of this are now shown in Figures 2 and 8. Redoing this on the Ne_NoTr it is apparent here that there indeed is an apparent altitude/range dependence. The reason for this is unclear and should be the topic of future investigations. If there are remaining structures in the final density-estimates from the fits of the ion-line spectrum/ACF after best effort calibrations and corrections, this method can still be used to remove or at least reduce such structures, regardless of how they bleed into the data-product.

We have added a discussion on this in the conclusions section, where we also suggest a future study on this topic. Essentially, the recommended strategy for calibration of a multibeam ISR in the future would be one of these two options:

1. Flat field Ne_NoTr to estimate relative correction factors for different beams, and then perform the fitting.
2. Flat field N_e only when it is sufficiently large, so that the non-linear Debye length correction is sufficiently small. Apply the resulting correction factor when redoing the fitting from scratch.

However, we think this step is beyond the scope of this article, which identifies the issue and shows a simple method for obtaining correction factors to the magic constant.