Review of “Improving the Magic Constant - Data-based Calibration of Phased Array Radars”

This manuscript addresses a significant problem with current phased array incoherent scatter radars, namely the relative calibration issues across the field of view. These issues are not widely appreciated by end users of ISR data products who are not phased array experts, and fixing these issues has significant implications for how ISR data is used to study ionospheric density gradients and irregularities. The two methods presented in this paper are useful improvements over the methodology for producing calibrated AMISR data. Nonetheless, the paper has a number of technical gaps that require further discussion and examination.

Issues

1. Equation 3 in the manuscript does not match the established definition of K_{sys} used in the rest of the AMISR literature. The correct equation is Eq. 16 of *Vadas and Nicolls* (2009).

$$P_{Rx} = P_{Tx}\frac{\tau_p K_{sys}}{R^2}\frac{N_e}{(1+k^2\lambda_{De}^2)\left(1+k^2\lambda_{De}^2+\frac{T_e}{T_i}\right)} \tag{1}$$

   In this equation, τ_p is the pulse length in seconds, which is missing from equation 3 in this manuscript. The k in this equation is the radar’s Bragg wavenumber, and the electron Debye length is given by

$$\lambda_{De}^2 = \frac{\epsilon_0 k_B T_e}{e^2 N_e}. \tag{2}$$

2. This manuscript does not address the nonlinearity in ISR measurements. The nonlinear effects are small for very large densities, but they become progressively more important as $k^2\lambda_{De}^2$ becomes larger. The nonlinearities manifest in two ways. First, the correct way to solve for electron density from received power is to first define

$$\alpha = N_e^{-1} \tag{3}$$

$$\beta = \frac{k^2\epsilon_0 k_B T_e}{e^2} \tag{4}$$

$$\gamma = \frac{\tau_p K_{sys} P_{Tx}}{R^2 P_{Rx}}, \tag{5}$$

then rearrange equation 1 as a cubic in α

$$\alpha\left(1+\beta\alpha\right)\left(1+\frac{T_e}{T_i}+\beta\alpha\right)=\gamma. \tag{6}$$

Solving this cubic for α yields 3 roots, but normally only one of them is positive and real. The electron density is the inverse of that root. For RISR-N's frequency of 442.5 MHz and a typical daytime F-region temperature of $T_e = 2800$ K, $\beta = 4.6 \times 10^9$ m^{-3}. The closer the electron density is to β, the more severe the nonlinear finite Debye length corrections become.

The second source of nonlinearity in ISR theory has to do with the fitting for T_e itself. One needs a reasonably good guess for the absolute calibration in order to fit the spectrum for T_e and T_i. The shape of the spectrum depends on the value of $k^2\lambda_{De}^2$, which means that changing the electron density does not simply scale the spectrum up and down. This is easiest to see if you write the ion-line ISR spectrum in the form of *Farley* (1966)

$$S\left(\omega\right)=N_e r_e^2\frac{\left|y_e\right|^2\Re\{y_i\}}{\left|y_e+\frac{T_e}{T_i}y_i-ik^2\lambda_{De}^2\right|^2}\frac{d\omega}{\pi\omega} \tag{7}$$

$$y=-i+\theta\int_0^\infty \exp\left[i\theta t-\phi^{-2}\sin^2\alpha\sin^2\frac{1}{2}\phi t-\frac{t^2}{4}\cos^2\alpha\right]dt \tag{8}$$

$$\theta=\frac{\omega}{k}\sqrt{\frac{m}{2k_BT}} \tag{9}$$

$$\psi=\frac{\Omega}{k}\sqrt{\frac{m}{2k_BT}} \tag{10}$$

The admittances y_e and y_i are only functions of temperatures, but the term $-ik^2\lambda_{De}^2$ that explicitly appears in the denominator creates nonlinear dependence of the shape of the spectrum on the absolute electron density. This is a subtle effect for small $k^2\lambda_{De}^2$, but it becomes progressively more important at smaller densities. This $-ik^2\lambda_{De}^2$ is essentially an extra damping of ion acoustic waves due to finite Debye length corrections. If you underestimate N_e, then you will also systematically overestimate T_e/T_i. If you fit ISR data with a very bad calibration coefficient, then the fitted temperatures will be noticeably biased.

The correct procedure for dealing with these nonlinearities is to iteratively re-estimate K_{sys} and re-do the nonlinear fitting with the new K_{sys} to propagate all the nonlinear corrections through the system.

The discussion on line 344 about biases during low density measurements has completely missed how the ISR theory becomes progressively more nonlinear at lower electron density.

3. The manuscript's discussion of "Darkfield" estimation for ISR is perplexing and does not demonstrate a deep understanding of the low-level radar signal processing. The

statement on line 136 that there is no way to make a Darkfield measurement for a radar is absolutely false. All AMISR experiments are designed to measure the noise levels on every beam and every interleaved frequency channel (each frequency channel done separately!) by taking dedicated noise samples from data windows corresponding to extremely long effective ranges. Furthermore, the end of the primary data windows from ranges above the top of the ionosphere are effectively a second estimate of the noise levels. This noise background is analogous to the "Darkfield" for a camera, and it is not at all small. It usually bigger than the ionospheric signal itself, and the processing already subtracts it out.

With this in mind, it is unclear what η in Equation 3 is supposed to represent. Is this supposed to be a residual noise level that is left over after noise subtraction due to a relative bias between the noise samples and the true noise level? If the noise sampling is working correctly, the expected value of η should already be zero.

The explanation on line 165 of choosing $\bar{N_e^{Df}} = 10^9$ m^{-3} does not make much sense. The value of 10^9 m^{-3} might as well be zero. The "Darkfield" level η cannot be in units of electron density. If η is non-zero due to a failure of noise subtraction, then the derived electron density increases like R^2 with increasing altitude. This error, when it occasionally happens, is very obvious.

The sentence on line 349-352 also makes no sense. The first part of the sentence is about the zero lag, and the end of the sentence is about the long lags. The noise estimation and removal process is not done as a scalar noise subtraction. AMISR estimates the entire noise ACF from the noise window samples, and the shapes of these measured noise ACFs are consistent with the autocorrelation if the impulse response of the final FIR filter in the receiver. These noise ACFs are subtracted from the entire measured ACF, lag-by-lag. If this was not already done correctly, the AMISR temperature estimates would be completely screwed up.

4. The manuscript needs to discuss issues of altitude interpolation more carefully, especially for the flat fielding. The data are naturally organized by range, and then gated into altitude gates for fitting the ACFs. The fitted data from different beams always comes out at slightly different altitudes. Before flat fielding, how exactly are the data from different beams being interpolated to a single altitude for flat fielding? Is this nearest neighbor, linear interpolation, or something else? Are the flat fielding results affected by different interpolation methods?

5. Why are the white lines in Figure 7 jagged lines? The terminator should be smoothly varying boundary as a function of time and magnetic latitude.

6. The discussion of possible reasons for an apparent range dependence to the corrections has missed one of the most obvious sources of error: correlations between fitted density and fitted temperatures. The entire analysis assumes the fitted T_e/T_i is known and correct. In reality the fitted T_e/T_i and its errors are altitude dependent, and the joint posterior distribution of N_e and T_e/T_i is likely varying with altitude along each beam. This raises the question of whether the flat fielding should be applied to the uncorrected electron density (Ne_NoTr) rather than the fitted electron density.

References

Farley, D. T. (1966), A theory of incoherent scattering of radio waves by a plasma: 4. the effect of unequal ion and electron temperatures, *Journal of Geophysical Research (1896-1977)*, *71*(17), 4091–4098, doi:https://doi.org/10.1029/JZ071i017p04091.

Vadas, S. L., and M. J. Nicolls (2009), Temporal evolution of neutral, thermospheric winds and plasma response using pfisr measurements of gravity waves, *Journal of Atmospheric and Solar-Terrestrial Physics*, *71*(6), 744–770, doi:https://doi.org/10.1016/j.jastp.2009.01.011.